Answers - Circles: circumference and area

Year: Year 8

Answer to Question 1

a. The diameter is twice the radius: $d = 2 \times 7 =$**14 cm**.
b. $C = \pi d = \pi \times 14 = 43.98\ldots$, which is **44.0 cm** to 1 d.p.
c. $C = 2\pi r = 2 \times \pi \times 7 = 44.0$ cm. Both formulae agree because $2r$ is the diameter.
d. Leave π as a symbol and write the number in front: 14π **cm**, which is the exact answer.

Answer to Question 2

a. $C = 2\pi r = 2 \times \pi \times 3 = 18.85\ldots$, so **18.8 cm**.
b. With a diameter use $C = \pi d = \pi \times 20 =$**62.8 cm**.
c. $C = 2 \times \pi \times 4.5 = 28.27\ldots$, so **28.3 m**.
d. $C = \pi \times 12.6 = 39.58\ldots$, so **39.6 mm**.

Estimate check: each circumference is just over three times the diameter, for example $3 \times 20 = 60$ for part (b).

Answer to Question 3

The area formula is $A = \pi r^2$: square the radius first, then multiply by π. When a diameter is given, halve it to find the radius.

a. $A = \pi \times 5^2 = \pi \times 25 =$ **78.5 cm²**
b. $A = \pi \times 2.5^2 = \pi \times 6.25 =$ **19.6 m²**
c. radius $= 7$, $A = \pi \times 49 =$ **153.9 cm²**
d. radius $= 4.5$, $A = \pi \times 20.25 =$ **63.6 mm²**

The units of area are always squared: cm², m² or mm².

Answer to Question 4

Answers in terms of π are exact: do the arithmetic with the numbers and leave π as a symbol at the end.

a. $\pi \times 36 = 36\pi$ **cm²**
b. $2\pi \times 6 = 12\pi$ **cm**
c. radius 5, $\pi \times 25 = 25\pi$ **m²**
d. $\pi \times 10 = 10\pi$ **m**

Answer to Question 5

To work backwards, put the known value into the formula and use inverse operations: divide by π, then halve or take the square root as needed.

a. $d = C \div \pi = 31.4 \div 3.14 =$ **10 cm**, so $r =$ **5 cm**
b. $d = 62.8 \div 3.14 = 20$, so $r =$ **10 m**
c. $r^2 = 78.5 \div 3.14 = 25$, so $r =$ **5 cm**
d. $r^2 = 49$, $r = 7$, so $d =$ **14 cm**

Answer to Question 6

a. $\frac{1}{2} \times \pi \times 10^2 = 50\pi =$ **157.1 cm²**
b. Curved edge $= \frac{1}{2} \times 2\pi \times 10 = 10\pi = 31.4$. Straight edge $= 20$. Perimeter = **51.4 cm**
c. $\frac{1}{4} \times \pi \times 100 = 25\pi =$ **78.5 cm²**
d. Curved edge $= \frac{1}{4} \times 2\pi \times 10 = 5\pi = 15.7$. Two radii $= 20$. Perimeter = **35.7 cm**

A common mistake is to forget the straight edges: the perimeter of a semicircle or quarter circle includes the diameter or the two radii.

Answer to Question 7

a. $C = \pi d = \pi \times 66 = 207.34\ldots$, so **207.3 cm**.
b. One turn moves the bike one circumference: 207.3 cm ÷ 100 = **2.07 m**.
c. 1 km = 1 000 m. $1000 \div 2.0735 = 482.3$, so about **482 complete turns**.
d. 3.2 km = 3 200 m. $3200 \div 2.0735 = 1543.3$, which is about **1 500 turns** to the nearest 100.

Answer to Question 8

a. Small: $\pi \times 10^2 = 314$ cm². Medium: $\pi \times 14^2 = 616$ cm². Large: $\pi \times 18^2 = 1018$ cm².
b. Small: $600 \div 314.2 = 1.91$p. Medium: $950 \div 615.8 = 1.54$p. Large: $1400 \div 1017.9 = 1.38$p.
c. The **large** pizza has the lowest cost per cm² (about 1.38p), so it is the best value.
d. No. The area of the large pizza is $1018 \div 314 = 3.24$ times the small one, so it is about **3.2 times** as big, not 2 times. Doubling the diameter multiplies the area by 4.

Answer to Question 9

a. The radius is half the diameter: $3 \div 2 =$**1.5 m**.
b. $A = \pi \times 1.5^2 = \pi \times 2.25 =$**7.07 m²**.
c. The path adds 1 m all round: $1.5 + 1 =$**2.5 m**.
d. Path = large circle − pond: $\pi \times 2.5^2 - \pi \times 1.5^2 = 19.63 - 7.07 =$**12.6 m²**.
e. $12.57 \times 18 = 226.19$, so the paving costs about **£226**.

Answer to Question 10

a. The two semicircles join to make a circle whose diameter is the width of the pitch: **90 m**.
b. $C = \pi d = \pi \times 90 =$**282.7 m** for both curved ends together.
c. Add the two straights: $2 \times 100 + 282.7 = 482.7$, so about **483 m** per lap.
d. 5 km = 5 000 m. $5000 \div 483 = 10.4$, so she runs **10 complete laps** and needs part of an 11th lap to reach 5 km.

Answer to Question 11

a. The cloth hangs 15 cm over on each side: $80 + 2 \times 15 =$**110 cm**.
b. Radius 55 cm: $\pi \times 55^2 = \pi \times 3025 =$**9 503 cm²**.
c. Divide by 10 000 to change to m²: 0.9503 m². Cost $0.9503 \times 12 =$**£11.40**.
d. The lace goes round the circumference: $\pi \times 110 = 345.6$, so **346 cm** of lace.

Answer to Question 12

a. The semicircle sits on the 120 cm width, so its diameter is 120 cm and its radius is **60 cm**.
b. $120 \times 70 =$**8 400 cm²**.
c. Half of a circle of radius 60: $\frac{1}{2} \times \pi \times 60^2 =$**5 654.9 cm²**.
d. $8400 + 5654.9 = 14054.9$ cm². Divide by 10 000: **1.41 m²** of glass.
e. The frame goes up both sides ($2 \times 70 = 140$ cm) and round the curved top ($\pi \times 60 = 188.5$ cm): 328.5 cm = 3.285 m. Cost $3.285 \times 35 =$**£114.97**.

Answer to Question 13

a. $C = \pi \times 7.4 = 23.25\ldots$, so **23.2 cm**.
b. Unrolled, the label is a rectangle 23.25 cm by 10.2 cm: $23.25 \times 10.2 = 237.1$, so **237 cm²**.
c. Radius $7.4 \div 2 = 3.7$ cm: $\pi \times 3.7^2 =$**43.0 cm²**.
d. 237 cm² is more than 200 cm², so the company's claim is **true**.

Answer to Question 14

a. In one hour the tip goes once round a circle of radius 1.2 m: $2\pi \times 1.2 = 7.54$, so **7.5 m**.
b. 15 minutes is a quarter of an hour: $7.54 \div 4 =$**1.88 m**.
c. $\pi \times 1.5^2 = 7.07$, so **7.1 m²**.
d. 20 minutes is $\frac{20}{60} = \frac{1}{3}$ of a turn. Sector area $\frac{1}{3} \times \pi \times 1.2^2 =$**1.51 m²**.

Answer to Question 15

a. $\pi \times 8^2 =$ **201.1 m²**
b. $8 + 2 = 10$ m. Area $= \pi \times 100 =$ **314.2 m²**
c. $314.2 - 201.1 =$ **113.1 m²**
d. Seed: $201.1 \times 1.40 = 281.49$. Flowers: $113.1 \times 6.50 = 735.13$. Total $= 1016.62$, so about **£1017**
e. Half the lawn is 100.5 m². The border is 113.1 m², which is bigger, so he is **not** right.

The border is a ring, so its area is the difference between the two circles, not the area of a circle of radius 2 m.

Answer to Question 16

a. $\pi \times 12^2 =$ **452 cm²**
b. $21^2 =$ **441 cm²**
c. Tin A: 452 cm², which is at least 450, so it is big enough. Tin B: 441 cm², which is less than 450, so it is **not** big enough.
d. Tin A: $\pi \times 24 = 75.4$ cm. Tin B: $4 \times 21 = 84$ cm. Tin **B** needs the longer ribbon (84 cm against 75.4 cm).

Answer to Question 17

a. The sector angle out of the full turn: $\frac{60}{360} = \frac{1}{6}$.
b. $\pi \times 12^2 = \pi \times 144 =$**452.4 cm²**.
c. One sixth of the circle: $452.4 \div 6 =$**75.4 cm²**.
d. Full circumference $2\pi \times 12 = 75.4$ cm; one sixth is **12.6 cm**.
e. The perimeter is the arc plus the two radii: $12.6 + 12 + 12 =$**36.6 cm**.

Answer to Question 18

a. Back: $\pi \times 1.8 =$ **5.65 m**. Front: $\pi \times 0.9 =$ **2.83 m**.
b. $100 \div 5.6549 =$ **17.7 turns**
c. $100 \div 2.8274 =$ **35.4 turns**
d. The front wheel makes twice as many turns. Its diameter is half as big, so its circumference is half as big, so it must turn twice as often to cover the same distance.

Answer to Question 19

a. $30 \times 20 =$**600 m²**.
b. Diameter 20 m, so radius 10 m: $\frac{1}{2} \times \pi \times 10^2 =$**157.1 m²**.
c. $600 + 157.1 =$**757.1 m²**.
d. $757.1 \div 8 = 94.6$, and we must have enough, so round up to **95 litres**.
e. $95 \div 5 = 19$ tins exactly: $19 \times 22 =$**£418**.

Answer to Question 20

a. $(2r)^2 = 4r^2$
b. πr^2
c. $\frac{\pi r^2}{4r^2} = \frac{\pi}{4} =$ **0.785**, so about 78.5%.
d. $4 \times 36 - \pi \times 36 = 144 - 113.1 =$ **30.9 cm²**
e. Four circles: $4\pi \times 36 = 452.4$ cm². Fraction $= \frac{452.4}{576} = 0.785$, which is the same $\frac{\pi}{4}$. Cutting more small circles does not waste less.

The fraction $\frac{\pi}{4}$ does not depend on r, so every circle cut from its smallest surrounding square wastes about 21.5% of the metal.