Test answers - Algebra: expressions and factorising

Year: Year 8 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. Only like terms can be combined. The a terms give $5a - 2a = 3a$ and the b terms give $3b + 4b = 7b$. The answer is $3a + 7b$. (1 mark for each correct term)
b. Multiply each term inside the bracket by 3: $3 \times 2x = 6x$ and $3 \times (-7) = -21$. The answer is $6x - 21$. (1 mark)
c. The highest common factor of 8 and 12 is 4. Write 4 outside the bracket and divide each term by 4: $8p \div 4 = 2p$ and $12 \div 4 = 3$. The answer is $4(2p + 3)$. (1 mark)

Check (c) by expanding: $4 \times 2p + 4 \times 3 = 8p + 12$. An answer such as $2(4p + 6)$ is correct but not fully factorised.

Answer to Question 2 (4 marks)

a. Expand both brackets: $4(x + 3) = 4x + 12$ and $2(x - 1) = 2x - 2$ (1 mark). Collect the x terms, $4x + 2x = 6x$, and the numbers, $12 - 2 = 10$: $6x + 10$ (1 mark).
b. The minus sign in front of the second bracket multiplies both of its terms: $-3 \times y = -3y$ and $-3 \times (-4) = +12$. So $5(2y - 1) - 3(y - 4) = 10y - 5 - 3y + 12$ (1 mark for expanding with correct signs). Collect: $10y - 3y = 7y$ and $-5 + 12 = 7$, giving $7y + 7$ (1 mark).

Check (b) with $y = 1$: the original gives $5 \times 1 - 3 \times (-3) = 5 + 9 = 14$, and $7 \times 1 + 7 = 14$. The answer can also be written $7(y + 1)$.

Answer to Question 3 (4 marks)

a. Substitute, keeping the negative value in brackets: $2 \times 4 - 3 \times (-3) = 8 - (-9) = 8 + 9 =$**17**. (1 mark)
b. $a^2 = 16$ and $b^2 = (-3)^2 = 9$, because a negative number squared is positive. So $16 + 9 =$**25**. (1 mark)
c. Look for the highest common factor of each part. The numbers 12 and 18 share 6; both terms contain at least one x and at least one y. The HCF is $6xy$. Dividing: $12x^2y \div 6xy = 2x$ and $18xy^2 \div 6xy = 3y$. The answer is $6xy(2x + 3y)$. (1 mark for a correct partial factor, 1 mark for the full HCF)

Check (c) by expanding: $6xy \times 2x + 6xy \times 3y = 12x^2y + 18xy^2$.

Answer to Question 4 (4 marks)

a. The bus costs £60 plus $5n$ for n pupils, and the snacks cost $3n$ (1 mark). Total cost $= 60 + 5n + 3n = 60 + 8n$ pounds (1 mark).
b. Substitute $n = 24$: $60 + 8 \times 24 = 60 + 192 =$ **£252**. (1 mark)
c. The cost must not be more than £400: $60 + 8n \leq 400$. Subtract 60: $8n \leq 340$. Divide by 8: $n \leq 42.5$. Pupils come in whole numbers, so round down to **42 pupils**. (1 mark)

42 pupils cost $60 + 8 \times 42 = £396$, while 43 would cost £404, which is over the budget. Rounding up here would break the budget, so this is a case where 42.5 must be rounded down.

Answer to Question 5 (4 marks)

a. Multiplying n by 3 gives $3n$; adding 12 gives $3n + 12$; dividing by 3 gives $\frac{3n + 12}{3}$ (1 mark). Divide each term on top by 3: $\frac{3n}{3} + \frac{12}{3} = n + 4$ (1 mark).
b. The simplified expression $n + 4$ means "the starting number plus 4" for every value of n, so Hana is **correct** (1 mark). An example illustrates it: with $n = 10$, $3 \times 10 + 12 = 42$ and $42 \div 3 = 14 = 10 + 4$ (1 mark).

One example on its own is not a proof, since it shows only a single case. The algebra in part (a) is what proves the claim for every number, including negatives and decimals.