Lesson - Algebra: expressions and factorising

Year: Year 8

This lesson covers the language of algebra, collecting like terms, the index laws for letters, expanding brackets and factorising. Read each box in order and try every worked example before you read the solution.

Starter activity

Think of a number, double it, add 6, then halve your answer. Try it with 4, 10 and 25. What do you notice about each answer compared with the starting number? Use a letter n for the number to explain why this always works.

The language of algebra

Definition: term and expression

A **term** is a single part of an expression, such as $3x$ or -5. An **expression** is a collection of terms with no equals sign, for example $4x + 3y - 7$. The sign in front of a term belongs to that term.

Definition: like terms and coefficient

Like terms have exactly the same letters raised to the same powers, so $3x$ and $5x$ are like terms but $3x$ and $3x^2$ are not. The number in front of the letters is the **coefficient**: in $3x$ the coefficient is 3.

Simplifying by collecting like terms

Rule: collecting like terms

Add or subtract the coefficients of like terms and keep the letter part unchanged. Unlike terms cannot be combined.

Worked example 1: collecting like terms

Simplify $4x + 3y - x + 2y$.

Group the like terms: $(4x - x) + (3y + 2y)$. Then $4x - x = 3x$ and $3y + 2y = 5y$, so the answer is $3x + 5y$.

Multiplying and dividing terms

Key fact: multiplying and dividing terms

- Multiply the numbers, then the letters: $3a \times 4b = 12ab$.
- Powers of the same letter: $x^2 \times x^3 = x^5$ (add the indices).
- Division: $12x^5 \div 3x^2 = 4x^3$ (divide the numbers, subtract the indices).
- Power of a power: $(2a^3)^2 = 2^2 \times a^6 = 4a^6$. The number is squared too.

Worked example 2: index laws with letters

Simplify $\frac{15a^4b}{5a^2}$ and $(3x^2)^3$.

Divide the numbers: $15 \div 5 = 3$. Subtract the indices of a: $a^4 \div a^2 = a^2$. The b stays. So $\frac{15a^4b}{5a^2} = 3a^2b$.

For the second part, cube the number and multiply the index: $(3x^2)^3 = 3^3 \times x^6 = 27x^6$.

Expanding brackets

Definition: expanding

To **expand** (or multiply out) a bracket, multiply every term inside by the term outside.

Key fact: expanding with numbers and letters

$3(x + 4) = 3x + 12$ and $5(2a - 3) = 10a - 15$. When the outside term is a letter, use the index laws: $x(x + 5) = x^2 + 5x$ and $2x(3x - 4) = 6x^2 - 8x$.

Method: expand and simplify

1. Multiply each term inside the first bracket by the term outside, including its sign.
2. Do the same for the next bracket. A minus sign in front means multiply by a negative number.
3. Collect like terms.

Worked example 3: expand and simplify

Expand and simplify $4(2x - 1) - 3(x - 2)$.

Expand the first bracket: $8x - 4$.
Expand the second bracket, multiplying by -3: $-3x + 6$.

Combine like terms: $8x - 4 - 3x + 6 = 5x + 2$.

Watch out: minus signs

A very common error is writing $-3x - 6$ instead of $-3x + 6$. Multiply -3 by -2 and you get $+6$.

Factorising

Definition: factorising

Factorising is the reverse of expanding. You write an expression as a product by taking out the **highest common factor** (HCF) of all the terms and placing the rest inside brackets.

Method: factorising

1. Find the HCF of the numbers.
2. Find the letters that appear in every term, using the smallest power of each.
3. Write the HCF outside the bracket and divide each term by it to fill the bracket.
4. Check by expanding.

Worked example 4: common number factor

Factorise $6x + 9$.

The HCF of 6 and 9 is 3. Dividing each term by 3 gives $2x$ and 3. So $6x + 9 = 3(2x + 3)$.

Worked example 5: letters and numbers

Factorise $12x^2 - 8x$.

The HCF of 12 and 8 is 4. Both terms contain x, and the smallest power is x^1, so the HCF is $4x$. Then $12x^2 \div 4x = 3x$ and $8x \div 4x = 2$. So $12x^2 - 8x = 4x(3x - 2)$.

Check by expanding: $4x \times 3x = 12x^2$ and $4x \times (-2) = -8x$. It matches.

Worked example 6: two letters

Factorise $6ab + 9a^2$.

The HCF is $3a$. $6ab \div 3a = 2b$ and $9a^2 \div 3a = 3a$. Answer: $3a(2b + 3a)$.

Substitution, checking and identities

Key fact: checking by substitution

You can check any simplification by substituting a number into both versions. If the values agree, the working is probably correct. Choose a number that is not 0 or 1 to avoid accidental matches.

Worked example 7: checking

Show that $3(x+2)+2(x-1)$ simplifies to $5x+4$ by trying $x=3$.

The original gives $3\times5+2\times2=15+4=19$. The answer gives $5\times3+4=19$. The values agree, so the simplification is correct.

Definition: identity

An **identity** is true for every value of the letter, unlike an equation, which is true only for some values. To show an identity, expand one side and compare. For example $3(2x+1)+3=6x+6=6(x+1)$. Some books write $\equiv$ for an identity.

Worked example 8: algebra in context

A rectangle has sides $(x+3)$ cm and $(2x-1)$ cm. Find its perimeter in its simplest form.

Perimeter $=2\times[(x+3)+(2x-1)]=2(3x+2)=6x+4$ cm. Algebra describes the perimeter for every value of x in one line.

Watch out: common mistakes

- Multiplying only the first term in a bracket: $3(x+4)=3x+4$ is wrong.
- Collecting unlike terms, such as $3x+2y=5xy$.
- Sign errors when a minus sign is in front of a bracket.
- Not taking out the *highest* common factor: $12x^2-8x=2(6x^2-4x)$ is not fully factorised.
- Writing $x^2\times x^3=x^6$; add the indices, do not multiply.

Worked example 9: algebra in a shop

A notebook costs £n and a pen costs £p. Emma buys 3 notebooks and 4 pens and pays with a £20 note. Write an expression for her change and find it when $n=2.50$ and $p=1.20$.

Total cost $=3n+4p$. Change $=20-(3n+4p)=20-3n-4p$. With the numbers: $3\times2.50+4\times1.20=7.50+4.80=12.30$, so her change is $20-12.30=£7.70$.

Worked example 10: factorise fully

Factorise fully $18a^2b - 27ab^2$.

Find the HCF of the numbers: 9 (the largest number that divides both 18 and 27). Find the HCF of the letters: a appears as a^2 and a, so take a; b appears as b and b^2, so take b. The HCF is $9ab$. Divide each term: $18a^2b \div 9ab = 2a$ and $27ab^2 \div 9ab = 3b$. So $18a^2b - 27ab^2 = 9ab(2a - 3b)$. Expand to check: $9ab \times 2a = 18a^2b$ and $9ab \times 3b = 27ab^2$.

Worked example 11: expanding with a negative factor

Expand and simplify $7 - 3(x - 2)$.

Be careful: the minus sign belongs to the 3. Expand $-3(x - 2) = -3x + 6$. Then $7 - 3x + 6 = 13 - 3x$. A common error is to write $7 - 3x - 6$, which gives the wrong answer $1 - 3x$.

Worked example 12: substituting into a formula

The cost in pounds of hiring a bike is $C = 8 + 3h$, where h is the number of hours. Find the cost for 5 hours, and the number of hours that costs £26.

For 5 hours: $C = 8 + 3 \times 5 = 8 + 15 = £23$. For £26: $8 + 3h = 26$, so $3h = 18$ and $h = 6$ hours. Substituting finds an output; solving an equation finds an input.

Note: how to choose the right tool

Look at the question word. If it says *simplify*, collect like terms. If it says *expand*, multiply out the brackets. If it says *factorise*, do the reverse and put the brackets back. If it says *work out the value* or *substitute*, replace each letter by its number. Writing the instruction word at the top of your working helps you to choose the correct method and avoid losing easy marks.

Watch out: signs and powers

- $3x + 4x = 7x$, but $3x \times 4x = 12x^2$. Adding changes the coefficient only; multiplying also changes the power.
- $(2x)^2 = 4x^2$, not $2x^2$: the bracket squares the 2 as well.
- When you factorise, always check by expanding. If you do not recover the original expression, something has gone wrong.

Method: proving a number trick

1. Call the starting number n.
2. Write an expression after every step of the trick.

3. Simplify the final expression.
4. If the letter n disappears, the answer is the same for every starting number.

Did you know?

The word *algebra* comes from the Arabic *al-jabr*, from a book written around the year 820 by the mathematician al-Khwarizmi in Baghdad. Today algebra is used by programmers, engineers and scientists. A spreadsheet in a shop uses algebra every time it applies a formula such as $B2 \times C2$ to find a total price.

Putting the skills together

Expanding and factorising are two directions of the same road: expanding removes brackets, factorising puts them back. Being fluent in both lets you rewrite an expression in whichever form is most useful. A factorised expression shows its factors clearly, which helps when you divide or solve equations, while an expanded expression is easier to add to other expressions or to substitute into.

Example: choosing the useful form

Show that $4n + 6$ is always even for any whole number n.

Factorising gives $4n + 6 = 2(2n + 3)$. Because the expression is 2 multiplied by a whole number, it is a multiple of 2, so it is always even. In the expanded form this is hard to see; in the factorised form the proof takes one line. This kind of short algebraic argument is called a proof, and it is much stronger than testing a few numbers, because it works for every value of n at once.

Did you know?

The word “algebra” comes from the title of a book written in Baghdad around the year 820 by Muhammad ibn Musa al-Khwarizmi. His name also gave us the word “algorithm”, which today describes the step-by-step instructions that computers follow.

Otto's tip

Expand to check a factorised answer. If expanding your brackets brings you back to the start, you are right. Factorising and expanding are a pair of opposite moves, like going up and down a staircase.