Answers - Algebra: expressions and factorising

Year: Year 8

Answer to Question 1

a. $4 + 7 - 3 = 8$, so $8a$
b. x terms: $5x + 3x = 8x$. y terms: $2y - 6y = -4y$. Answer $8x - 4y$
c. p^2 terms: $8p^2 + 2p^2 = 10p^2$. p terms: $-3p + 9p = 6p$. Answer $10p^2 + 6p$
d. ab and ba are the same term, so $3ab + 4ba = 7ab$. Answer $7ab - 2a$

Parts (b), (c) and (d) still have two kinds of term. x and y (or p^2 and p, or ab and a) are different letters or powers, so they are not like terms and cannot be added.

Like terms have exactly the same letters and powers; only their coefficients are added or subtracted.

Answer to Question 2

Multiply or divide the numbers first, then the letters. When the same letter is multiplied, add the powers; when it is divided, subtract the powers.

a. $3 \times 5 = 15$, so $15ab$
b. $4 \times 2 = 8$ and $x \times x^2 = x^3$. Answer $8x^3$
c. $12 \div 4 = 3$ and $m^3 \div m = m^2$. Answer $3m^2$
d. $18 \div 6 = 3$ and p cancels. Answer $3q$
e. $(2y)^3 = 2y \times 2y \times 2y = 8y^3$. Answer $8y^3$

Answer to Question 3

Multiply the term outside the bracket by each term inside, keeping the signs: a negative outside changes the sign of every term.

a. $4 \times x + 4 \times 6 = 4x + 24$
b. $3 \times 2a - 3 \times 5 = 6a - 15$
c. $-2 \times y + (-2) \times 7 = -2y - 14$
d. $x \times x - x \times 9 = x^2 - 9x$
e. $5p \times 2p + 5p \times 3q = 10p^2 + 15pq$

Check any answer by substituting a simple value, for example $x = 1$ in (a): $4 \times 7 = 28$ and $4 + 24 = 28$.

Answer to Question 4

Expand each bracket separately, paying special attention when a negative number multiplies a negative term, then collect the like terms.

a. $3x + 6 + 2x + 10 = 5x + 16$
b. $8a + 4 - 3a + 6 = 5a + 10$ (note that $-3 \times -2 = +6$)
c. $5y - 15 - 2y + 8 = 3y - 7$ (note that $-2 \times -4 = +8$)
d. $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

In (d) the two brackets are the same, which is why the result can also be written as $(x + 3)(x + 4)$.

Answer to Question 5

Find the highest common factor of the numbers and of the letters, write it outside the bracket, and divide each term by it to find what goes inside.

a. HCF is 3: $3(2x + 5)$
b. HCF is 4: $4(3a - 2)$
c. HCF is x: $x(x + 7)$
d. HCF is $2p$: $2p(5q - 2)$
e. HCF is $3mn$: $3mn(3m + 4n)$. Check: $3mn \times 3m = 9m^2n$ and $3mn \times 4n = 12mn^2$.

Expanding each answer gives back the original expression, which is the quickest check.

Answer to Question 6

a. $4 \times 3 + (-2) = 12 - 2 =$**10**.
b. $2 \times 5 - 3 \times (-2) = 10 - (-6) = 10 + 6 =$**16**.
c. $3^2 + (-2)^2 = 9 + 4 =$**13**; the square of a negative number is positive.
d. $ab = 3 \times (-2) = -6$, then $-6 + 5 = -1$.
e. Bracket first: $3 - (-2) = 5$, then $3 \times 5 =$**15**.

Putting negative values in brackets when substituting prevents sign errors.

Answer to Question 7

a. Hall: $30 + 8h$
b. Food: $4g$
c. $C = 30 + 8h + 4g$
d. $C = 30 + 8 \times 3 + 4 \times 15 = 30 + 24 + 60 =$ **£114**
e. $C = 30 + 8 \times 4 + 4 \times 20 = 30 + 32 + 80 = £142$. This is less than £150, so **yes**, with £8 to spare.

Keeping the hall hours and the guests as separate letters lets Maya test any combination quickly before booking.

Answer to Question 8

Perimeter adds the four sides; area multiplies length by width.

a. $P = x + (x+6) + x + (x+6) = 4x + 12$ metres
b. $A = x(x+6) = x^2 + 6x$ square metres
c. Perimeter $= 4 \times 4 + 12 = 28$ m. Cost $= 28 \times 12 =$ **£336**
d. $A = 4^2 + 6 \times 4 = 16 + 24 =$ **40 m²**

With $x = 4$ the allotment is 4 m by 10 m, so the perimeter is 28 m and the area 40 m², which agrees with the expressions.

Answer to Question 9

Each monthly cost is the fixed charge plus the cost per GB multiplied by the number of GB used.

a. $6 + 2d$
b. $10 + 1.5d$
c. Blue: $6 + 2 \times 8 = £22$. Red: $10 + 1.5 \times 8 = £22$. The costs are equal.
d. Blue: $6 + 20 = £26$. Red: $10 + 15 = £25$. **Red** is cheaper by **£1**.

Below 8 GB Blue Mobile is cheaper, at 8 GB they cost the same, and above 8 GB Red Mobile is cheaper, because its price per GB is lower.

Answer to Question 10

Lengths of rows and columns are found by multiplying the number of tiles by the tile side t; the area is width × length.

a. $8t$ cm
b. $5t$ cm
c. $8t \times 5t = 40t^2$ cm²
d. $40 \times 30^2 = 40 \times 900 = 36000$ cm². Since 1 m² = 10 000 cm², the area is **3.6 m²**.

Remember to convert square centimetres to square metres by dividing by 10 000, not by 100.

Answer to Question 11

a. $3p + 2r$
b. $5p + 4r$
c. $3p + 2r + 5p + 4r = 8p + 6r$
d. $8 \times 20 + 6 \times 35 = 160 + 210 =$ **370p = £3.70**
e. Josh minus Layla: $5p + 4r - (3p + 2r) = 2p + 2r$. With numbers: $2 \times 20 + 2 \times 35 = 40 + 70 =$ **110p**. Check: Josh $= 100 + 140 = 240$p, Layla $= 60 + 70 = 130$p, difference 110p.

Using algebra first shows that Josh pays for exactly two more pencils and two more rubbers than Layla.

Answer to Question 12

a. $6 + 4w$
b. $30 - 2w$
c. See the table below.
d. The table shows both have £22 when $w = 4$. Check: $6 + 4w = 30 - 2w \Rightarrow 6w = 24 \Rightarrow w = 4$. So **after 4 weeks**.

w	0	2	4	6
Tom $6 + 4w$	6	14	22	30
Ella $30 - 2w$	30	26	22	18

Answer to Question 13

a. Area of the patio = length × width: $4(3x + 2) = 12x + 8$ m².
b. The pond is x m by 5 m: $5x$ m².
c. Paved area = patio − pond: $12x + 8 - 5x = 7x + 8$ m².
d. With $x = 3$: patio $4 \times 11 = 44$ m², pond 15 m², paved $44 - 15 =$ **29 m²**, which matches $7 \times 3 + 8$.

Answer to Question 14

a. $4a + 6c$
b. Each ticket costs £1 less: $4(a - 1) + 6(c - 1)$
c. $4a - 4 + 6c - 6 = 4a + 6c - 10$
d. Without: $36 + 36 =$ **£72**. With: $72 - 10 =$ **£62**.

The discount is £1 on each of the 10 tickets, which is why the expanded expression ends in -10.

Answer to Question 15

a. $n \to n + 3 \to 4(n + 3) \to 4(n + 3) - 8 \to \dfrac{4(n + 3) - 8}{4} \to \dfrac{4(n + 3) - 8}{4} - n$
b. $\dfrac{4n + 12 - 8}{4} - n = \dfrac{4n + 4}{4} - n = n + 1 - n =$ **1**. The final answer is always 1.
c. $n = 7$: $10 \to 40 \to 32 \to 8 \to 8 - 7 = 1$. $n = -2$: $1 \to 4 \to -4 \to -1 \to -1 - (-2) = 1$. Both give 1.
d. $\dfrac{6(n + 3) - 8}{6} - n = \dfrac{6n + 10}{6} - n = \dfrac{10}{6} = \dfrac{5}{3}$. The letter still disappears, so the

answer is always the same, but this time it is $\frac{5}{3}$ rather than 1.

The number you start with always cancels out, so the final answer cannot depend on it: this is an algebraic proof that the trick always works.

Answer to Question 16

a. $3x + 12 - 2x + 2 = x + 14$
b. **Bea** is right.
c. Ali wrote $-2 \times -1 = -2$ and got $12 - 2 = 10$. Cal added $3x + 2x$ instead of subtracting. Dev did not multiply the -1 by -2 at all (he got $12 + 0$).
d. Original: $3 \times 9 - 2 \times 4 = 27 - 8 = 19$. Answer: $5 + 14 = 19$. They match.

Most mistakes in expanding come from the sign of the second bracket: $-2 \times (-1) = +2$.

Answer to Question 17

a. Add the two widths: $(x + 5) + x = 2x + 5$ cm.
b. The shape is one large rectangle $(2x + 5)$ by 6: $2(2x + 5) + 2 \times 6 = 4x + 10 + 12 = 4x + 22$ cm.
c. Area = width × height: $6(2x + 5) = 12x + 30$ cm².
d. $12x + 30 = 90$. Subtract 30: $12x = 60$. Divide by 12: $x = 5$. Check: the shape is 15 cm by 6 cm, and $15 \times 6 = 90$.

Answer to Question 18

a. Monday: x boxes of 6 and y boxes of 4: $6x + 4y$ doughnuts.
b. Tuesday: $6 \times 2x + 4(y + 3) = 12x + 4y + 12$, so $12x + 4y + 12$.
c. Add the two days and collect: $6x + 12x = 18x$, $4y + 4y = 8y$: $18x + 8y + 12$.
d. $18 \times 5 + 8 \times 8 + 12 = 90 + 64 + 12 =$ **166** doughnuts. Check: Monday $30 + 32 = 62$, Tuesday $60 + 44 = 104$, total 166.

Answer to Question 19

Each adult ticket costs £5x and each child ticket £3x, so multiply by the number of tickets and collect like terms.

a. $2 \times 5x + 3 \times 3x = 10x + 9x = 19x$
b. $19 \times 2 =$ **£38**
c. $19x = 57 \Rightarrow x = 57 \div 19 =$ **3**
d. $5xa + 3xc = x(5a + 3c)$ after taking out the common factor x.

Check (c): with $x = 3$, adults pay £15 and children £9, and

$2 \times 15 + 3 \times 9 = 30 + 27 = 57$.

Answer to Question 20

a. $(x + 2) + (x + 7) + (x + 6) = 3x + 15$
b. Middle row: $(x + 9) + (x + 5) + (x + 1) = 3x + 15$. First column: $(x + 2) + (x + 9) + (x + 4) = 3x + 15$. Yes, the same.
c. Diagonal 1: $(x + 2) + (x + 5) + (x + 8) = 3x + 15$. Diagonal 2: $(x + 6) + (x + 5) + (x + 4) = 3x + 15$. Both match, so it is a magic square.
d. $3x + 15 = 33 \Rightarrow 3x = 18 \Rightarrow x = 6$. The square is 8, 13, 12 / 15, 11, 7 / 10, 9, 14. Every line adds up to 33.

The numbers added to x are 1 to 9 arranged as an ordinary magic square whose lines total 15, which is why every row, column and diagonal gives the same expression $3x + 15$.