Lesson - Solving one-step and two-step equations

Year: Year 7

An equation says two things are equal. Solving an equation means finding the value of the letter that makes it true. In this chapter you will use inverse operations and the balance method.

Starter activity

Mia thinks of a number, doubles it and adds 5. She gets 17. Can you work out her number by "undoing" each step in reverse order? Compare your method with a partner and say what you notice about the order of the steps.

What is an equation?

Definition: equation

An **equation** is a statement that two expressions are equal, such as $x + 3 = 10$. A **solution** is the value of the letter that makes the equation true. Here $x = 7$.

Definition: unknown

The letter in an equation is called the **unknown**.

Inverse operations

Key fact: inverse operations

Addition and subtraction are inverses. Multiplication and division are inverses. To undo an operation, do the inverse to both sides.

Rule: balance method

An equation is like a balance. Whatever you do to one side, you must do to the other, so it stays balanced.

One-step equations

Example 1

Solve $x + 8 = 15$.

Subtract 8 from both sides: $x = 7$.

Example 2

Solve $x - 4 = 9$ and $5x = 30$.

Add 4: $x = 13$. Divide by 5: $x = 6$.

Example 3

Solve $\frac{x}{3} = 7$.

Multiply both sides by 3: $x = 21$.

Two-step equations

Method: solving a two-step equation

1. Undo the addition or subtraction first.
2. Then undo the multiplication or division.
3. Do the same to both sides each time.
4. Check by substituting your answer.

Example 4

Solve $3x + 5 = 26$.

Subtract 5: $3x = 21$. Divide by 3: $x = 7$. Check: $3 \times 7 + 5 = 26$. Correct.

Example 5

Solve $\frac{x}{4} - 2 = 3$.

Add 2: $\frac{x}{4} = 5$. Multiply by 4: $x = 20$.

Equations with negative answers

Example 6

Solve $2x + 9 = 3$.

Subtract 9: $2x = -6$. Divide by 2: $x = -3$. Check: $2 \times (-3) + 9 = 3$.

Watch out

Do not be afraid of negative answers. Also remember to do the same thing to both sides: subtracting 9 from one side only gives the wrong answer.

Equations with brackets

Method: solving an equation with brackets

1. Expand the bracket.
2. Collect terms.
3. Solve as a two-step equation.

Example 7

Solve $4(x-3)=20$.

Divide both sides by 4: $x-3=5$, so $x=8$. Or expand: $4x-12=20$, $4x=32$, $x=8$.

Unknowns on both sides

Rule: unknowns on both sides

Collect the letters on one side by adding or subtracting the smaller letter term from both sides. Then solve as usual.

Example 8

Solve $5x-3=2x+12$.

Subtract $2x$: $3x-3=12$. Add 3: $3x=15$. Divide: $x=5$.

Forming and solving equations

Example 9

I think of a number, multiply it by 3 and add 4. The answer is 19. What is my number?

$3x+4=19$, so $3x=15$ and $x=5$.

Example 10

The perimeter of a rectangle with sides x and $x+3$ is 26 cm. Find x.

$2x+2(x+3)=26$, so $4x+6=26$, $4x=20$ and $x=5$.

A worked word problem

A taxi costs £4 plus £2.50 per mile. A journey costs £16.50. How many miles was it? The equation is $2.5m + 4 = 16.5$. Subtract 4: $2.5m = 12.5$. Divide by 2.5: $m = 5$. The journey was 5 miles.

Solving with fractions as answers

Example 11

Solve $4x = 10$ and $3x + 2 = 6$.

$x = \frac{10}{4} = \frac{5}{2}$ or 2.5. For the second, $3x = 4$, so $x = \frac{4}{3}$ or $1\frac{1}{3}$.

Note

Leave answers as fractions when they do not divide exactly. Do not round unless the question asks you to.

Checking and using equations

An equation can model a real situation. Define the unknown, write the equation from the words, solve it, and then answer in a full sentence with units.

Example 12

Ben is 5 years older than Cara. Their ages add up to 35. How old is Cara?

Let Cara be c. Ben is $c + 5$. $c + c + 5 = 35$, so $2c = 30$ and $c = 15$. Cara is 15 and Ben is 20.

Example 13

Solve $7 - 2x = 1$.

Subtract 7: $-2x = -6$. Divide by -2: $x = 3$. Check: $7 - 2 \times 3 = 1$.

Example 14

Solve $3(2x + 1) = 2(x + 9)$.

Expand: $6x + 3 = 2x + 18$. Subtract $2x$: $4x + 3 = 18$. Subtract 3: $4x = 15$. So $x = 3.75$.

Note

The solution is the one value that makes both sides equal. If a check does not work, look again at the signs.

Common mistakes

- Doing different things to each side.
- Undoing in the wrong order (multiply/divide before add/subtract).
- Sign mistakes with negatives.
- Not checking the answer by substitution.

Worked example in context

A plumber charges £40 call-out plus £25 per hour. A bill is £140. How many hours did the job take?

Equation: $40 + 25h = 140$. Subtract 40: $25h = 100$. Divide by 25: $h = 4$. The job took 4 hours.

Common mistakes

- Doing the same step to only one side of the equation.
- Adding where you should subtract: the inverse of +6 is -6.
- Forgetting to multiply every term when there are brackets.

Did you know?

The equals sign was invented in 1557 by a Welsh mathematician called Robert Recorde. He chose two parallel lines because, he said, "no two things can be more equal". Before then, people wrote the word "aequales" in full.

Starter activity: the balance scales

Picture a pair of old-fashioned kitchen scales. On the left pan there are three identical bags of sweets and two loose sweets; on the right pan there are seventeen loose sweets, and the scales balance. How many sweets are in each bag? If you take two loose sweets off each side, the scales still balance, leaving three bags against fifteen sweets. Sharing the fifteen sweets equally between the three bags shows that each bag holds five. This is exactly what we do when we solve $3x + 2 = 17$: whatever we do to one side, we do to the other, so the equation stays balanced.

Method: solving an equation with brackets and unknowns on both sides

1. Expand any brackets, multiplying every term inside.
2. Collect the letter terms on the side where there are more of them, by adding or subtracting the same term on both sides.
3. Collect the number terms on the other side.
4. Divide both sides by the number in front of the letter, then substitute your answer into the original equation to check it.

Example 1

Solve $4(x-3)=2x+8$.

First we expand the bracket: $4x-12=2x+8$. There are more x's on the left, so we subtract $2x$ from both sides to get $2x-12=8$. Next we add 12 to both sides, which gives $2x=20$. Finally we divide both sides by 2, so $x=10$. Checking: the left side is $4(10-3)=28$ and the right side is $2\times10+8=28$. Both sides are equal, so the solution is correct.

Example 2

Solve $\frac{x}{3}+5=9$.

The last thing that happened to x was "add 5", so we undo it first by subtracting 5 from both sides: $\frac{x}{3}=4$. Now x has been divided by 3, so we multiply both sides by 3 to get $x=12$. Check: $12\div3+5=4+5=9$. Undoing the operations in the reverse order is like taking off your shoes before your socks: the last thing done is the first thing undone.

Example 3

The angles in a triangle are x, $2x$ and $x+40$ degrees. Find each angle.

The angles of a triangle add up to 180°, so we form the equation $x+2x+x+40=180$. Collecting like terms gives $4x+40=180$. Subtracting 40 from both sides gives $4x=140$, and dividing by 4 gives $x=35$. The angles are 35°, 70° and 75°. They add up to 180°, which confirms the answer. Many word problems become easy once you have written the right equation.

Example 4

A gym charges a £20 joining fee plus £15 per month. Another gym charges £35 per month and no joining fee. After how many months do the two gyms cost the same?

Let m be the number of months. The first gym costs $20+15m$ pounds and the second costs $35m$ pounds. Setting them equal gives $20+15m=35m$. Subtracting $15m$ from both sides gives $20=20m$, so $m=1$. After one month both gyms cost £35; after that the first gym is cheaper because it charges less each month.

Watch out

An answer to an equation does not have to be a whole number or positive. $2x=7$ gives $x=3.5$, and $x+9=4$ gives $x=-5$. Do not change the equation to make the answer "nicer". Leave fractional answers as fractions or exact decimals unless the question asks you to round.

Did you know?

The equals sign = was invented by the Welsh mathematician Robert Recorde in 1557. He chose two parallel lines of the same length “because no two things can be more equal”. Before that, mathematicians wrote the words “is equal to” in full every time, which made long calculations very slow to write and to read. His book, *The Whetstone of Witte*, was also one of the first in English to use the plus and minus signs that we still write in every lesson today.

Otto's tip

Think of a balance. Do the same to both sides, and always check your answer by putting it back in!