Lesson - Sequences

Year: Year 7

A sequence is a list of numbers that follows a rule. In this chapter you will continue sequences, use term-to-term and position-to-term rules and find the nth term of arithmetic sequences.

Starter activity

Draw the first three patterns of the following: pattern 1 is one square, pattern 2 is a 2 × 2 square, pattern 3 is a 3 × 3 square. Count the squares in each. What would pattern 10 contain? Describe the rule you used and say whether the differences are constant.

What is a sequence?

Definition: sequence and term

A **sequence** is an ordered list of numbers. Each number is a **term**. The first term is written T_1, the second T_2, and so on. The nth term is the term in position n.

Term-to-term rules

Definition: term-to-term rule

A **term-to-term rule** tells you how to get from one term to the next, such as “add 3” or “multiply by 2”.

Example 1

The first term is 4 and the rule is “add 5”. Write the first five terms.

4, 9, 14, 19, 24.

Example 2

Find the rule and the next two terms of 2, 6, 18, 54.

Each term is multiplied by 3 (a geometric sequence). The next terms are $54 \times 3 = 162$ and $162 \times 3 = 486$.

Position-to-term rules and the nth term

Rule: position-to-term rule

A **position-to-term rule** (the nth term) gives each term directly from its position n. Replace n with the position number.

Example 3

The nth term is $4n - 1$. Find the first three terms and the 20th term.

$n = 1$: 3. $n = 2$: 7. $n = 3$: 11. The 20th term is $4 \times 20 - 1 = 79$.

Finding the nth term of an arithmetic sequence

Definition: arithmetic sequence

An **arithmetic sequence** goes up or down by the same amount each time. This amount is the **common difference**.

Rule: nth term of an arithmetic sequence

The nth term is $dn + c$, where d is the common difference and $c = T_1 - d$ is the number you add or subtract to match the first term.

Method: finding the nth term

1. Find the common difference d.
2. Write down dn.
3. Compare dn with the sequence to find c (the zero term).
4. Write the nth term as $dn + c$ and check with the first two terms.

Example 4

Find the nth term of 5, 8, 11, 14, ...

The difference is 3, so start with $3n$: 3, 6, 9, 12. Add 2 to get 5, 8, 11, 14. The nth term is $3n + 2$.

Example 5

Find the nth term of 20, 17, 14, 11, ...

The difference is -3, so start with $-3n$: $-3, -6, -9, -12$. Add 23 to get 20, 17, 14, 11. The nth term is $23 - 3n$.

Is a number in the sequence?

Example 6

Is 100 in the sequence with nth term $3n + 2$?

Solve $3n + 2 = 100$: $3n = 98$, so $n = 32.67$. This is not a whole number, so 100 is not in the sequence. (Is 101? $3n = 99$ so $n = 33$. Yes.)

Watch out

The position n must be a positive whole number. If you get a fraction, the number is not in the sequence.

Special sequences

Key fact: special sequences

Square numbers: 1, 4, 9, 16, 25 (nth term n^2). Triangle numbers: 1, 3, 6, 10, 15 (add one more each time). Cube numbers: 1, 8, 27, 64. Fibonacci: 1, 1, 2, 3, 5, 8, 13 (add the two previous terms). Powers of 2: 2, 4, 8, 16, 32.

Example 7

Find the next two terms of 1, 1, 2, 3, 5, 8, ...

Each term is the sum of the two before it. $5 + 8 = 13$ and $8 + 13 = 21$.

Patterns with shapes

Example 8

Pattern 1 uses 4 matchsticks, pattern 2 uses 7 and pattern 3 uses 10. Find the nth term and the number of matchsticks in pattern 12.

The difference is 3, so $3n + 1$. Pattern 12: $3 \times 12 + 1 = 37$.

Note

The coefficient of n is the number of new sticks added each time, and the constant is the number the pattern starts with.

Common mistakes

- Using $n + d$ instead of $dn + c$.
- Forgetting to check the nth term with the first term.
- Confusing the term-to-term rule with the position-to-term rule.
- Using the first term as the constant (it should be $T_1 - d$).

Worked example in context

A plant is 8 cm tall and grows 3 cm each week. How tall is it after 12 weeks?

The sequence is 11, 14, 17, ... with nth term $3n + 8$. After 12 weeks the height is $3 \times 12 + 8 = 44$ cm.

Common mistakes

- Writing the nth term as $n + 3$ when the difference is 3: the coefficient of n must equal the common difference.
- Forgetting that the first term goes with n = 1, not n = 0.
- Assuming every sequence adds the same amount each time.

Did you know?

The Fibonacci sequence appears in nature: the number of petals on many flowers (3, 5, 8, 13, 21) and the spirals of a sunflower head follow it. It is named after the Italian mathematician Fibonacci who described it about 800 years ago when thinking about rabbits.

Starter activity: growing patterns

Take some counters or draw dots. Make a row of 3 dots, then a row of 5, then a row of 7. Before drawing the next one, predict how many dots it will have and explain how you know. Then ask a harder question: how many dots would the 50th row have? Counting on 2 each time would take a long time, so we need a rule that links the position of a term directly to its value. Finding that rule, the nth term, is the main skill of this chapter.

Method: deciding whether a number is in a sequence

1. Write the nth term of the sequence, for example $5n + 3$.
2. Set it equal to the number you are testing, for example $5n + 3 = 78$.
3. Solve the equation: $5n = 75$, so $n = 15$.
4. If n is a positive whole number, the number is in the sequence and n tells you its position. If n is a fraction or a decimal, the number is not in the sequence.

Example 1

Find the nth term of the sequence 9, 15, 21, 27, ...

First we find the common difference: $15 - 9 = 6$, $21 - 15 = 6$ and $27 - 21 = 6$. The terms go up by 6 each time, so the nth term starts with $6n$. The sequence $6n$ would be 6, 12, 18, 24, ... Each term of our sequence is 3 more than this, so the nth term is $6n + 3$. Check with the 4th term: $6 \times 4 + 3 = 27$, which matches.

Example 2

The nth term of a sequence is $40 - 3n$. Find the first negative term.

The sequence starts $37, 34, 31, \ldots$ and goes down by 3 each time. We want $40 - 3n$, which means $3n > 40$, so $n > 13.3$. The first whole number greater than 13.3 is 14, and the 14th term is $40 - 42 = -2$. Check the 13th term: $40 - 39 = 1$, which is still positive. So the first negative term is -2, in position 14.

Example 3

A sequence starts 2, 6, 18, 54. Is it arithmetic or geometric? Find the next two terms.

The differences are 4, 12 and 36, which are not equal, so the sequence is not arithmetic (not linear). Dividing each term by the one before gives $6 \div 2 = 3$, $18 \div 6 = 3$ and $54 \div 18 = 3$. Each term is the previous one multiplied by 3, so the sequence is geometric. The next two terms are $54 \times 3 = 162$ and $162 \times 3 = 486$. Geometric sequences grow much faster than linear ones.

Note

A term-to-term rule tells you how to get from one term to the next, so it is good for continuing a sequence a few steps. A position-to-term rule, the nth term, tells you how to get any term straight from its position, so it is much better for finding the 100th or the 1 000th term without writing out all the terms before it.

Watch out

The number in front of n is the common difference, but the number added on is *not* the first term. For 5, 8, 11, ... the nth term is $3n + 2$, not $3n + 5$. Always check your rule by substituting $n = 1$ and $n = 2$ and comparing with the first two terms of the sequence.

Did you know?

The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, ... was described by Leonardo of Pisa in 1202 in a puzzle about breeding rabbits. Its numbers turn up again and again in nature: the number of petals on many flowers and the number of spirals on a pine cone or a pineapple are very often Fibonacci numbers. If you divide one term by the term before it, the answer gets closer and closer to the golden ratio, about 1.618.

Otto's tip

Test your nth term with n = 1 and n = 2. If it gives the first two terms, you have got it right!