Answers - Sequences

Year: Year 7

Answer to Question 1

First look at how each term changes: a constant difference means add or subtract, a constant multiplier means multiply, and other patterns may be special sequences such as square or Fibonacci numbers.

a. 24, 29, 34. Add 5 each time.
b. 26, 20, 14. Subtract 6 each time.
c. 162, 486, 1458. Multiply by 3 each time.
d. 25, 36, 49. The square numbers.
e. 13, 21, 34. Add the two previous terms (Fibonacci).

Answer to Question 2

a. The difference is 3, so start with $3n$ (3, 6, 9, ...). The terms are 2 more: $3n + 2$.
b. Difference 5: $5n$ gives 5, 10, 15, ... The terms are 3 less: $5n - 3$.
c. Difference -3: $-3n$ gives $-3, -6, -9, \ldots$ The terms are 23 more: $-3n + 23$ (or $23 - 3n$).
d. Difference 4: $4n$ gives 4, 8, 12, ... The terms are 3 less: $4n - 3$.
e. Difference 2: $2n$ gives 2, 4, 6, ... The terms are 5 more: $2n + 5$.

Check each rule with $n = 1$: for example $3 \times 1 + 2 = 5$, the first term of (a).

Answer to Question 3

Substitute the position number for n to find a term. To test whether a number is in the sequence, set $4n + 3$ equal to it and check whether n is a whole number.

a. 7, 11, 15, 19.
b. $4 \times 20 + 3 =$**83**.
c. $4 \times 100 + 3 =$**403**.
d. $4n + 3 = 83$, $4n = 80$, $n = 20$. **Yes**, it is the 20th term.
e. $4n + 3 = 50$, $4n = 47$, $n = 11.75$ is not a whole number. **No**.

Answer to Question 4

a. 3, 5, 9, 17, 33.
b. The differences are 2, 4, 8, 16, which are not constant, so it is **not linear**.
c. 2, 8, 14, 20.
d. $6n - 4 = 50$, $6n = 54$, $n =$**9**.

A sequence is linear only when the difference between consecutive terms stays the same; here the differences double each time.

Answer to Question 5

a. Square numbers are $n \times n$: **1, 4, 9, 16, 25, 36**.
b. Triangular numbers add 2, then 3, then 4, ...: **1, 3, 6, 10, 15**.
c. Cube numbers are $n \times n \times n$: **1, 8, 27, 64, 125**.
d. Each power of 2 doubles the previous one: **2, 4, 8, 16, 32, 64**.
e. $49 = 7^2$, $64 = 8^2$ and $100 = 10^2$ are square. 50 and 120 lie between squares, so the square numbers are **49, 64 and 100**.

Answer to Question 6

a. Three steps to go from 3 to 18: difference $15 \div 3 = 5$. **8, 13**.
b. Difference 6: **8** and **26**.
c. Four steps from 50 to 10: difference $-40 \div 4 = -10$. **40, 30, 20**.
d. Four steps from 11 to 31: difference 5. First term $11 - 10 =$**1**.

Answer to Question 7

a. The amount goes up by £3 each week: week 4 is **£14** and week 5 is **£17**.
b. Difference 3, so start with $3n$ (3, 6, 9, ...). Each amount is 2 more: $3n + 2$ pounds.
c. Substitute $n = 12$: $3 \times 12 + 2 =$**£38**.
d. $3n + 2 > 50$ gives $3n > 48$, so $n > 16$. Week 16 is exactly £50, so the first week with more than £50 is **week 17** (£53).

Answer to Question 8

a. Each new house adds 4 matchsticks: $13 + 4 =$**17**.
b. Difference 4, so start with $4n$. For $n = 1$, $4n = 4$ but we need 5, so add 1: $4n + 1$.
c. Substitute $n = 30$: $4 \times 30 + 1 =$**121** matchsticks.
d. $4n + 1 \leq 150$, so $4n \leq 149$ and $n \leq 37.25$. She can build **37 houses** using $4 \times 37 + 1 = 149$ matchsticks, with **1** left over.

Answer to Question 9

a. Add 2 for each row: 12, 14, 16, 18, 20. Row 5 has **20** seats.
b. Difference 2, so $2n$; for $n = 1$ we need 12, so add 10: $2n + 10$.
c. $2n + 10 = 40$, so $2n = 30$ and $n =$**15**.
d. The back row is row 15 with **40** seats. The rows go from 12 to 40, with an

average of $\frac{12+40}{2} = 26$ seats, so the total is $15 \times 26 =$**390 seats**.

Answer to Question 10

a. The costs go 4, 6.50, 9, 11.50: each is £2.50 more than the last, which matches the rule.
b. Difference 2.5, so $2.5n$; for $n = 1$ this gives 2.5 but the cost is 4, so add 1.5: $C = 2.5n + 1.5$.
c. $2.5 \times 10 + 1.5 = 25 + 1.5 =$**£26.50**.
d. $2.5n + 1.5 = 34$, so $2.5n = 32.5$ and $n = 32.5 \div 2.5 =$**13 miles**.

Answer to Question 11

a. Year 3: 48. Year 4: 96.
b. No, it is **geometric**. The differences (6, 12, 24, 48) are not constant.
c. Year 5: 192. Year 6: **384**.
d. Year 7: 768. Year 8: 1 536. After **8 years**.

Doubling sequences are geometric: they grow slowly at first and then very quickly, which is why the rabbits pass 1 000 only two years after reaching about 400.

Answer to Question 12

The total number of tins for n rows is a triangular number, found by adding $1 + 2 + 3 + \cdots + n$.

a. $1 + 2 + 3 + 4 + 5 =$**15**.
b. **8**.
c. 1, 3, 6, 10, 15 (the triangular numbers).
d. 13 rows use 91 tins and 14 rows use 105 tins, so **13 rows** with **9 tins** left over.

Answer to Question 13

a. Add 7 cm each week: **19, 26, 33 and 40 cm**.
b. Start height 12 cm plus 7 cm for each of the n weeks: $7n + 12$.
c. $7 \times 10 + 12 =$**82 cm**.
d. 2 m = 200 cm. $7n + 12 > 200$ gives $7n > 188$ and $n > 26.86$. The first whole week is **week 27** (201 cm).

Answer to Question 14

a. 4, 5.5, 7, 8.5, 10 km.
b. $1.5n + 2.5$.
c. $1.5n + 2.5 > 10$, $n > 5$. Week **6** (11.5 km).
d. $1.5 \times 12 + 2.5 = 20.5$, so she runs 20.5 km, which is more than 20 km. The coach is **approximately right**.

Remember that “more than 10 km” rules out week 5, where the distance is exactly 10 km.

Answer to Question 15

To match a linear sequence with its rule, look at the difference (the number in front of n) and check the first term. A sequence that multiplies by the same number is a power rule.

a. A: $4n - 1$. B: $2n + 1$. C: $3n$. D: 3^n.
b. A: 39. B: 21. C: 30. D: 59 049.
c. D has the largest 10th term ($3^{10} = 59049$).
d. Difference 6 and first term 3: $6n - 3$.

Answer to Question 16

a. Black tiles are square numbers: pattern 4 has $4^2 =$**16**. White tiles go up by 4: $16 + 4 =$**20**.
b. White tiles: 8, 12, 16, 20, difference 4, so $4n$ plus 4: $4n + 4$.
c. Black tiles form an n by n square: n^2.
d. $4n + 4 = 36$, so $4n = 32$ and $n = 8$. Pattern **8** has $8^2 =$**64** black tiles.

Answer to Question 17

a. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.
b. $1 + 1 + 2 + 3 + 5 = 12$. The 7th term is 13 and $13 - 1 = 12$. They are equal.
c. 2, 8, 34, 144 (every third term is even).
d. 2, 5, 7, 12, 19, 31, 50, 81.

Answer to Question 18

a. The difference is 3, so the rule starts with 3n. $7 - 3 = 4$, so the nth term is **3n + 4**.
b. $3 \times 10 - 2 =$**28**. Beth subtracted first.
c. $4n + 5 = 31$, $4n = 26$, $n = 6.5$. Carl divided by 2 instead of 4. 31 is **not** in the sequence.
d. The sequence doubles each time, so the next term is **32**.

Answer to Question 19

a. Ali: 20, 25, 30, 35, 40, 45. Bea: 5, 13, 21, 29, 37, 45.
b. Ali: $20 + 5n$. Bea: $5 + 8n$.
c. $20 + 5n = 5 + 8n$, $15 = 3n$, $n =$**5** weeks (both have £45).
d. After 10 weeks: Ali £70, Bea £85. Bea has **£15** more.

Bea saves more each week, so although she starts with less, her total overtakes Ali's after week 5.

Answer to Question 20

a. Differences: 3, 5, 7. They are not constant, so the sequence is **not linear**.
b. Second differences: 2, 2.
c. Constant second differences of 2 tell you the nth term contains n^2 (the coefficient is half of 2).
d. $n^2 + 2 = 51$, $n^2 = 49$, $n = 7$. Pattern **7**.

Check: $7^2 + 2 = 49 + 2 = 51$.