Lesson - Ratio and proportion

Year: Year 7

Ratio and proportion compare quantities. In this chapter you will simplify ratios, share amounts, use the unitary method and solve problems with scale.

Starter activity

You and a friend share a bag of 30 sweets. You say you should get 2 sweets for every 1 sweet your friend gets. How many sweets does each of you get? What if the ratio was 3 : 2? Explain how you worked it out.

What is a ratio?

Definition: ratio

A **ratio** compares two or more quantities. The ratio of boys to girls being $3 : 2$ means for every 3 boys there are 2 girls. The order matters, and the ratio has no units.

Example 1

There are 12 red and 8 blue counters. Write the ratio of red to blue.

$12 : 8$.

Simplifying ratios

Rule: simplifying ratios

Divide every part of the ratio by a common factor, ideally the highest one. If the parts are in different units, convert them first.

Example 2

Simplify $12 : 8$ and $2.5 : 10$.

$12 : 8 = 3 : 2$, dividing by 4. For $2.5 : 10$, multiply by 2 to get $5 : 20$, then divide by 5 to get $1 : 4$.

Example 3

Write 40 cm to 1.2 m as a ratio in its simplest form.

Convert to cm: $40 : 120 = 1 : 3$.

Sharing in a given ratio

Method: sharing in a ratio

1. Add the parts of the ratio to find the total number of parts.
2. Divide the amount by the total to find the value of one part.
3. Multiply by each part to find each share.
4. Check the shares add up to the total.

Example 4

Share £60 in the ratio $2:3$.

$2+3=5$ parts. $60\div 5=12$. Shares: $2\times 12=$£24 and $3\times 12=$£36. Check: $24+36=60$.

Example 5

Sam, Tara and Uma share 45 sweets in the ratio $2:3:4$. How many does Tara get?

$2+3+4=9$ parts and $45\div 9=5$. Tara gets $3\times 5=15$ sweets.

Using one known quantity

Example 6

The ratio of cats to dogs is $3:5$. There are 15 cats. How many dogs are there?

3 parts = 15, so 1 part = 5. Dogs: $5\times 5=25$.

Watch out

Remember, when you use a part-to-part ratio you need to add the parts to find the whole. A ratio of $2:3$ means $\frac{2}{5}$ and $\frac{3}{5}$ of the total.

Proportion and the unitary method

Definition: direct proportion

Two quantities are in **direct proportion** if they increase or decrease in the same ratio. If one doubles, the other doubles.

Rule: unitary method

Find the value of one unit by dividing, then multiply to find the value you need.

Example 7

5 notebooks cost £7.50. Find the cost of 8 notebooks.

One notebook costs $7.50 \div 5 =$£1.50. Eight cost $8 \times 1.50 =$£12.

Example 8

A recipe for 4 people uses 300 g of flour. How much flour is needed for 10 people?

For 1 person: $300 \div 4 = 75$ g. For 10 people: $75 \times 10 = 750$ g.

Best buys

Method: finding the best buy

1. Find the cost per unit (per gram or per item) for each option.
2. Or find the amount per pound.
3. Compare and choose the cheaper per unit.

Example 9

A 500 g jar of honey costs £3.20 and a 750 g jar costs £4.50. Which is better value?

500 g: $3.20 \div 500 = 0.0064$ pounds per gram. 750 g: $4.50 \div 750 = 0.006$. The 750 g jar is better value.

Scale and maps

Definition: scale

A **scale** is a ratio that shows how a drawing compares with real life. A map scale of $1 : 50\,000$ means 1 cm on the map is 50 000 cm (500 m) on the ground.

Example 10

On a map with scale $1 : 25\,000$, two towns are 6 cm apart. How far apart are they in kilometres?

$6 \times 25\,000 = 150\,000$ cm. This is 1 500 m, or 1.5 km.

Note

To change cm to km, divide by 100 000 (cm to m is $\div 100$, and m to km is $\div 1000$).

Common mistakes

- Writing a ratio in the wrong order.
- Using the amount as one part: you must divide by the total number of parts.
- Not converting to the same units before simplifying.
- Adding the same number to both parts of a ratio. Ratios are scaled by multiplying.

Worked example in context

Blue and yellow paint are mixed in the ratio 3 : 5 to make 24 litres of green paint. How much of each is used?

Total parts: $3 + 5 = 8$. One part: $24 \div 8 = 3$ litres. Blue: $3 \times 3 = 9$ litres. Yellow: $5 \times 3 = 15$ litres.

Common mistakes

- Dividing the total by one of the ratio numbers instead of the total number of parts.
- Not using the same units on both sides of a ratio.
- Confusing the ratio 3 : 1 with the fraction one third (it is one quarter).

Did you know?

Artists and architects have used the "golden ratio" (about 1.6 : 1) for thousands of years because many people find it pleasing to the eye. Flags, credit cards and even paper sizes are designed using ratios.

Starter activity: making squash

A bottle of orange squash says “mix 1 part squash with 4 parts water”. Imagine using a small cup as your part: one cup of squash and four cups of water make five cups of drink. If you want twice as much, you use two cups of squash and eight cups of water, and the drink tastes exactly the same. This is the key idea of ratio: multiplying or dividing every part by the same number keeps the proportions, and therefore the taste, the colour or the shape, unchanged.

Method: ratio problems with a bar model

1. Draw one bar for each person or ingredient, split into the number of equal boxes given by the ratio.
2. Mark on the diagram the amount you know: it might be the total, one share, or

the difference between two shares.
3. Work out the value of one box by dividing the known amount by the number of boxes it covers.
4. Multiply to find every share you need, and check that the shares add up to the total.

Example 1

Kai and Lea share some stickers in the ratio 3 : 5. Lea gets 14 more stickers than Kai. How many stickers are there altogether?

Here we do not know the total; we know the difference. In the bar model Kai has 3 boxes and Lea has 5 boxes, so Lea has 2 boxes more than Kai. These 2 boxes are worth 14 stickers, so one box is $14 \div 2 = 7$ stickers. Kai has $3 \times 7 = 21$ and Lea has $5 \times 7 = 35$. Altogether there are $21 + 35 = 56$ stickers, which is 8 boxes of 7. Spotting what the known amount represents is the most important step.

Example 2

Write the ratio 1.2 m : 45 cm in its simplest form.

Before simplifying, both quantities must be in the same unit. 1.2 m is 120 cm, so the ratio is $120 : 45$. The highest common factor of 120 and 45 is 15, and dividing both numbers by 15 gives $8 : 3$. Once the units are the same they can be left out, because a ratio compares sizes and has no unit of its own.

Example 3

7 identical notebooks cost £9.80. How much do 12 notebooks cost?

We use the unitary method: first find the cost of one notebook, then multiply. One notebook costs $9.80 \div 7 = £1.40$. Twelve notebooks cost $12 \times 1.40 = £16.80$. A quick check: 12 is a bit less than twice 7, and £16.80 is a bit less than twice £9.80, so the answer is sensible.

Note

Two quantities are in direct proportion when doubling one doubles the other, tripling one triples the other, and so on. The cost of apples and the number of apples bought is a good example. Not every relationship works like this: a taxi fare with a fixed starting charge is not in direct proportion to the distance, because a journey of zero miles still costs money.

Watch out

The order of a ratio matters. If the ratio of boys to girls is 2 : 3, then there are *fewer* boys than girls, and the fraction of the class who are boys is $\frac{2}{5}$, not $\frac{2}{3}$. To change a

ratio into a fraction, always divide the part by the *total* number of parts.

Did you know?

Many artists and architects have used the “golden ratio”, about 1.618 : 1, because rectangles in this ratio are said to look especially pleasing. The same ratio appears in the spiral patterns of sunflower seeds and pine cones. Whether it really makes things more beautiful is still debated, but you can measure the sides of a credit card and see that it is quite close: about 8.6 cm by 5.4 cm, a ratio of roughly 1.59 : 1.

Otto's tip

Draw a bar model with the ratio parts. Label one part, and the rest is easy!