Lesson - Probability basics

Year: Year 7

Probability measures how likely something is. In this chapter you will use the probability scale, calculate simple probabilities and predict how often things happen.

Starter activity

Roll a dice 30 times and record how many times each number appears in a tally chart. Is each number equally likely? Compare your results with a partner and combine the data. Do the combined results look closer to what you expect from a fair dice?

What is probability?

Definition: probability, event and outcome

Probability measures how likely it is that something will happen. An **event** is something that may happen, such as rolling a 6. An **outcome** is one possible result.

Key fact: probability words

A probability of 0 means **impossible**. A probability of 1 means **certain**. $\frac{1}{2}$ means **even chance**. Probabilities are written as fractions, decimals or percentages between 0 and 1.

The probability scale

The scale runs from 0 (impossible) on the left, through $\frac{1}{2}$ (even chance), to 1 (certain) on the right. A fair coin lands on heads with probability $\frac{1}{2}$. The probability of rolling a 7 on a standard dice is 0. The probability that the sun rises tomorrow is 1.

Watch out

A probability can never be less than 0 or more than 1. Use fractions, decimals or percentages, not ratios such as 1 : 6.

Calculating probability

Rule: probability of equally likely outcomes

When all outcomes are equally likely,

$$P(\text{event}) = \frac{\text{number of outcomes that give the event}}{\text{total number of possible outcomes}}.$$

Example 1

A fair dice is rolled. Find the probability of a 4, an even number and a prime number.

$P(4) = \frac{1}{6}$. $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$ (2, 4, 6). $P(\text{prime}) = \frac{3}{6} = \frac{1}{2}$ (2, 3, 5).

Example 2

A bag has 3 red, 5 blue and 2 green counters. Find P(red), P(blue) and P(not green).



3 red, 5 blue, 2 green

There are 10 counters. $P(\text{red}) = \frac{3}{10}$, $P(\text{blue}) = \frac{5}{10} = \frac{1}{2}$ and $P(\text{not green}) = \frac{8}{10} = \frac{4}{5}$.

Probabilities add up to 1

Rule: complementary events

The probabilities of all possible outcomes add up to 1. So $P(\text{not } A) = 1 - P(A)$.

Example 3

The probability of rain is 0.3. What is the probability of no rain?

$1 - 0.3 = 0.7$.

Example 4

A spinner has $P(\text{red}) = 0.35$ and $P(\text{blue}) = 0.45$. The rest is green. Find $P(\text{green})$.

$1 - 0.35 - 0.45 = 0.2$.

Listing outcomes

Method: listing outcomes

1. List the outcomes in an organised way, such as a table or a list.

2. Count the total number of outcomes.
3. Count the outcomes that give the event.
4. Write the probability as a fraction.

Example 5

Two coins are tossed. Find the probability of one head and one tail.

The outcomes are HH, HT, TH, TT. There are 4. Two give one head and one tail, so the probability is $\frac{2}{4} = \frac{1}{2}$.

Example 6

Two dice are rolled. Find the probability that the sum is 7.

There are 36 outcomes. The sum 7 can be made in 6 ways: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). So the probability is $\frac{6}{36} = \frac{1}{6}$.

Expected outcomes and experiments

Rule: expected number

Expected number of times = probability $\times$ number of trials. The result of an experiment will not always match exactly.

Example 7

A fair dice is rolled 60 times. How many sixes do you expect?

$\frac{1}{6} \times 60 = 10$ sixes.

Definition: relative frequency

The **relative frequency** is the number of times an event happened divided by the number of trials. It estimates the probability when outcomes are not equally likely. More trials give a more reliable estimate.

Example 8

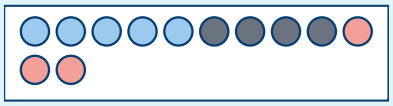
A drawing pin lands point up 37 times in 50 throws. Estimate the probability, and how many point-up landings you expect in 200 throws.

Relative frequency $= \frac{37}{50} = 0.74$. In 200 throws, $0.74 \times 200 = 148$.

Another worked example

Example 9

A box holds 12 pens: 5 blue, 4 black and 3 red. Find the probability that a pen taken at random is blue. Then find the probability that the next pen is red if the blue pen is not replaced.



5 blue, 4 black, 3 red

$P(\text{blue}) = \frac{5}{12}$. Black or red is the same as not blue: $\frac{7}{12}$. If the blue pen is not put back, there are 11 pens left, so $P(\text{red}) = \frac{3}{11}$. When items are not replaced, the total changes.

Note

Probability is useful in real life. Weather forecasts give the chance of rain as a percentage, and doctors use probabilities to describe the chance of side effects.

Common mistakes

- Saying the probability is $\frac{1}{2}$ just because there are two outcomes. They must be equally likely.
- Giving a probability above 1.
- Forgetting to simplify fractions.
- Thinking the next result must be different after a run of the same results. Each trial is independent.

Worked example in context

A bag has 3 red and 7 blue counters. What is the probability of picking a blue counter and how many blue counters would you expect in 50 picks (with replacement)?

P(blue) = $\frac{7}{10}$. Expected blue: $50 \times \frac{7}{10} = 35$.

Common mistakes

- Thinking that every event with two outcomes has probability 1/2.
- Writing probabilities greater than 1 or less than 0.
- Thinking that after a run of heads, tails is "due".

Did you know?

Probability began in France in the 1650s when two mathematicians, Pascal and Fermat, exchanged letters about how to share the winnings fairly in an unfinished game of dice. Today it is used in weather forecasts, insurance and in video games.

Starter activity: is the game fair?

Work with a partner. One of you scores a point when a single dice shows 1, 2, 3 or 4, and the other scores a point when it shows 5 or 6. Before you start, predict who will win after 30 rolls. Then play and record the results in a tally chart. You should find that the first player usually wins, because four of the six equally likely outcomes favour them. A game is fair only when every player has the same probability of winning, and probability is the tool that lets us check this before we play.

Method: calculating a theoretical probability

1. Check that all the outcomes are equally likely, for example a fair dice, a fair coin or an object chosen at random.
2. Count the total number of possible outcomes.
3. Count the number of outcomes that give the event you want.
4. Write the probability as a fraction: favourable outcomes over total outcomes, and simplify it if possible.

Example 1

The letters of the word STATISTICS are written on ten cards and one card is chosen at random. Find the probability that it is the letter S, and the probability that it is a vowel.

There are ten cards and each is equally likely to be chosen. The letter S appears three times (S, S, S), so P(S) $= \frac{3}{10}$. The vowels are A, I and I, which is three cards, so P(vowel) $= \frac{3}{10}$ as well. As a check, the probability of a consonant is $1 - \frac{3}{10} = \frac{7}{10}$, and indeed there are seven consonants: S, T, T, S, T, C, S.

Example 2

A drawing pin is dropped 200 times and lands point up 74 times. Estimate the probability that it lands point up, and predict how many times it will land point up in 500 drops.

A drawing pin is not symmetrical, so we cannot assume the two outcomes are equally likely. Instead we use the relative frequency from the experiment: $\frac{74}{200} = 0.37$. This is

our best estimate of the probability. For 500 drops we expect $500 \times 0.37 = 185$ landings point up. The more times an experiment is repeated, the more reliable the relative frequency becomes.

Example 3

A fair spinner has five equal sections coloured red, red, blue, green and yellow. It is spun twice. What is the probability of getting red both times?

One way is to list the outcomes in a sample space grid with 5 rows and 5 columns, which gives 25 equally likely outcomes. Red on the first spin can happen in 2 ways and red on the second in 2 ways, so there are $2 \times 2 = 4$ outcomes with red both times. The probability is $\frac{4}{25}$. In Year 8 you will learn to multiply the probabilities directly: $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$.

Watch out

Dice and coins have no memory. If a fair coin has landed on heads five times in a row, the probability of heads on the next flip is still $\frac{1}{2}$. Believing that tails is now "due" is called the gambler's fallacy, and it has cost many people a lot of money.

Did you know?

Probability theory began with a letter about a dice game. In 1654 a French gambler asked the mathematicians Blaise Pascal and Pierre de Fermat how to share the stakes fairly when a game is stopped early. Their letters to each other laid the foundations of probability, which is now used to forecast the weather, set insurance prices and test new medicines.

Otto's tip

Write the total number of outcomes on the bottom first. Then count the favourite outcomes carefully for the top. Your answer must lie between 0 and 1!