Answers - Probability basics

Year: Year 7

Answer to Question 1

a. A normal dice only has the numbers 1 to 6, all less than 7, so this will always happen: **certain** (probability 1).
b. Heads and tails are equally likely: **even chance** (probability 0.5).
c. February is only one month out of twelve: **unlikely** (about $\frac{1}{12}$).
d. There is no 7 on a normal dice: **impossible** (probability 0).
e. Rain at some point in a whole month is normal in the UK: **likely**, very close to certain but not completely.

Answer to Question 2

a. There are $5 + 3 + 2 = 10$ counters and 5 are red: $\frac{5}{10} = \frac{1}{2}$.
b. 3 of the 10 counters are blue: $\frac{3}{10}$.
c. P(green) $= \frac{2}{10}$, so P(not green) $= 1 - \frac{2}{10} = \frac{8}{10} = \frac{4}{5}$.
d. There are no yellow counters, so the event is impossible: **0**.
e. $\frac{1}{2} = 1 \div 2 =$ **0.5**, and $0.5 \times 100 =$ **50%**.

Answer to Question 3

a. The four probabilities must add up to 1: $1 - 0.3 - 0.25 - 0.2 =$ **0.25**.
b. $P(C) = 1 - 0.4 - 0.35 =$ **0.25**.
c. Being late and not being late cover every possibility, so P(not late) $= 1 - 0.12 =$ **0.88**.
d. Number of red = probability × total: $20 \times 0.35 =$ **7** red counters.

Answer to Question 4

a. Each coin can land H or T, so the outcomes are **HH, HT, TH, TT**: 4 equally likely outcomes.
b. Only HH gives two heads: $\frac{1}{4}$.

c. HT and TH both give one head and one tail: $\frac{2}{4} = \frac{1}{2}$.

d. "At least one head" means HH, HT or TH: $\frac{3}{4}$. Or $1 - P(TT) = 1 - \frac{1}{4}$.

Answer to Question 5

The grid shows all $6 \times 6 = 36$ equally likely totals. For each event, count the cells that match and write the count over 36, then simplify.

a. **36**.

b. $\frac{6}{36} = \frac{1}{6}$

c. Totals 10, 11, 12: $3 + 2 + 1 = 6$ outcomes, so $\frac{6}{36} = \frac{1}{6}$

d. Primes 2, 3, 5, 7, 11 occur $1 + 2 + 4 + 6 + 2 = 15$ times, so $\frac{15}{36} = \frac{5}{12}$.

Answer to Question 6

a. P(six) $= \frac{1}{6}$, so expect $\frac{1}{6} \times 120 =$ **20** sixes.

b. Expected number = probability × number of trials: $0.15 \times 200 =$ **30** late buses.

c. $0.3 \times 50 =$ **15** times blue.

d. Each spin or roll is random, so the results vary from one experiment to another. The expected number is what happens on average over many repeats, not a guarantee.

Answer to Question 7

There are $1 + 4 + 15 = 20$ prize-winning tickets out of 200 tickets, and every ticket is equally likely to be drawn.

a. $\frac{1}{200}$

b. $\frac{1 + 4 + 15}{200} = \frac{20}{200} = \frac{1}{10}$

c. $1 - 0.1 = 0.9$, so **90%**.

d. Dan has more tickets. Aisha expects $5 \times 0.1 = 0.5$ prizes and Dan expects $12 \times 0.1 = 1.2$ prizes, so **Dan** has the better chance.

Answer to Question 8

a. It either rains or it does not, so P(no rain on Saturday) $= 1 - 0.7 =$ **0.3**.

b. P(no rain on Sunday) $= 1 - 0.4 =$ **0.6**.
c. 0.7 is bigger than 0.4, so rain is more likely on **Saturday**.
d. Expected number = probability × trials: $20 \times 0.7 =$ **14** rainy Saturdays.

Answer to Question 9

a. Three of the eight sections show 3: $\frac{3}{8}$.
b. The even numbers are 2, 2 and 4: $\frac{3}{8}$.
c. Greater than 2 means 3, 3, 3, 4 or 5: $\frac{5}{8}$.
d. The primes are 2, 3 and 5 (1 and 4 are not prime). Sections 2, 2, 3, 3, 3, 5 make 6: $\frac{6}{8} = \frac{3}{4}$.

Answer to Question 10

a. $\frac{18}{30} = \frac{3}{5}$
b. $1 - \frac{4}{30} = \frac{13}{15}$
c. $28 \times 0.6 = 16.8$, so about **17 pupils**.
d. After the first pupil is chosen there are only 29 pupils left, and the numbers in each group may have changed.

In part (c) the probability $\frac{3}{5} = 0.6$ gives 16.8, which is not a whole number of pupils, so we round to about 17.

Answer to Question 11

a. 13 of the 52 cards are hearts: $\frac{13}{52} = \frac{1}{4}$.
b. There is one king in each suit, so 4 kings: $\frac{4}{52} = \frac{1}{13}$.
c. Hearts and diamonds are red, $2 \times 13 = 26$ cards: $\frac{26}{52} = \frac{1}{2}$.
d. There is only one king of hearts: $\frac{1}{52}$.
e. P(spade) $= \frac{1}{4}$, so P(not a spade) $= 1 - \frac{1}{4} = \frac{3}{4}$.

Answer to Question 12

a. Red $\frac{26}{80} = 0.325$, blue $\frac{18}{80} = 0.225$, green $\frac{22}{80} = 0.275$, yellow $\frac{14}{80} = 0.175$.
b. If fair, each colour has probability **0.25**.
c. The relative frequencies are close to 0.25 but not equal. With only 80 spins this could be chance, so more trials are needed.
d. $400 \times 0.325 =$**130** times.

Answer to Question 13

a. There are $6 + 4 + 2 = 12$ socks and 6 are black: $\frac{6}{12} = \frac{1}{2}$.
b. P(white) $= \frac{4}{12} = \frac{1}{3}$, so P(not white) $= \frac{2}{3}$.
c. After one black sock is removed, 11 socks remain and 5 are black: $\frac{5}{11}$.
d. 6 of the 12 socks are already black, so P(black) is exactly $\frac{1}{2}$. She should add **none**.

Answer to Question 14

a. $\frac{16}{40}$; divide by 8: $\frac{2}{5}$.
b. P(slow) $= \frac{4}{40} = \frac{1}{10}$, so P(not slow) $= \frac{9}{10}$.
c. P(dance) $= \frac{8}{40} = \frac{1}{5}$, so in 10 songs expect $10 \times \frac{1}{5} =$**2** dance songs.
d. $40 - 20 = 20$ songs are left, and $12 - 6 = 6$ of them are rock: $\frac{6}{20} = \frac{3}{10}$.

Answer to Question 15

a. Total 7 occurs 6 times out of 36: $\frac{1}{6}$.
b. Total 5 occurs 4 times and total 9 occurs 4 times: $\frac{8}{36} = \frac{2}{9}$.
c. $1 - \frac{6}{36} - \frac{8}{36} = \frac{22}{36} = \frac{11}{18}$.
d. The game is **not fair** because Ben has the higher probability ($\frac{8}{36} > \frac{6}{36}$). For a fair game Ben could win on totals of 2, 3 or 4 instead, which also has 6 outcomes

out of 36 (1 + 2 + 3).

Answer to Question 16

a. The three probabilities must add up to 1: $0.2 + x + 3x = 1$.
b. Collect: $4x + 0.2 = 1$, so $4x = 0.8$ and $x =$**0.2**. Then P(blue) = 0.2 and P(green) = 0.6.
c. Red $0.2 \times 40 =$**8**, blue $0.2 \times 40 =$**8**, green $0.6 \times 40 =$**24** (total 40).
d. Now 10 red out of 42 counters: $\frac{10}{42} = \frac{5}{21}$.

Answer to Question 17

a. Outcomes are not equally likely, so you cannot say 1/2. The probability is tiny.
b. Probabilities of mutually exclusive outcomes must add to at most 1 (0.6 + 0.5 = 1.1 is impossible if only one can happen).
c. Six rolls is too few to decide. The probability of no sixes in 6 rolls is about 33%.
d. $\frac{3}{3+5} = \frac{3}{8}$.

Answer to Question 18

a. Bag A: $\frac{4}{10} = 0.4$. Bag B: $\frac{3}{5} = 0.6$.
b. **Bag B**.
c. $\frac{7}{15}$, about **0.467**.
d. The average of 0.4 and 0.6 is 0.5. This is not the same because the bags have different sizes.

Combined bag: $4 + 3 = 7$ mints out of $10 + 5 = 15$ sweets. Bag A has more sweets, so its lower probability counts for more, pulling the combined value below 0.5.

Answer to Question 19

a. 2 coin results × 6 dice results = **12** equally likely outcomes, as in the table.
b. Heads with 2, 4 or 6: H2, H4, H6, 3 outcomes: $\frac{3}{12} = \frac{1}{4}$.
c. Tails with 5 or 6: T5, T6: $\frac{2}{12} = \frac{1}{6}$.
d. All 6 heads outcomes, plus T6 (H6 is already counted): $6 + 1 = 7$ outcomes, so $\frac{7}{12}$.

Answer to Question 20

a. 11 of the 12 months are different from the first pupil's: $\frac{11}{12}$.

b. Each new pupil must avoid the months already used:
$\frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} = \frac{11880}{20736} \approx$**0.573**.

c. "At least two share" is the opposite of "all different": $1 - 0.573 =$**0.427**.

d. **No**, about 43%. But with only 4 pupils and 12 months it is surprisingly close to a half; with 5 pupils it would be more than 60%.