Lesson - Perimeter, area and volume

Year: Year 7

Perimeter, area and volume describe the size of shapes. In this chapter you will calculate perimeter and area of rectangles, triangles, parallelograms and trapezia, and the volume of cuboids.

Starter activity

Measure the length and width of your classroom floor in steps or using a metre ruler. Work out the perimeter and the area. How many square tiles of side 50 cm would you need to cover the floor? Share your results and talk about why areas are measured in square units but perimeters in ordinary units.

Perimeter, area and volume

Definition: perimeter, area and volume

Perimeter is the distance around a shape, measured in cm or m. **Area** is the space inside a flat shape, in cm² or m². **Volume** is the space inside a 3D shape, in cm³ or m³.

Rectangles and squares

Rule: rectangle and square formulae

For a rectangle with length l and width w: $P = 2(l + w)$ and $A = lw$. For a square of side s: $P = 4s$ and $A = s^2$.

Example 1

A rectangle is 8 cm long and 5 cm wide. Find its perimeter and area.



$P = 2 \times (8 + 5) = 26$ cm. $A = 8 \times 5 = 40$ cm².

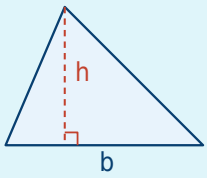
Triangles

Rule: area of a triangle

$A = \frac{1}{2} \times b \times h$, where b is the base and h is the perpendicular height (at right angles to the base). The perimeter is the sum of the three sides.

Example 2

Find the area of a triangle with base 10 cm and height 7 cm.



$A = \frac{1}{2} \times 10 \times 7 = 35$ cm².

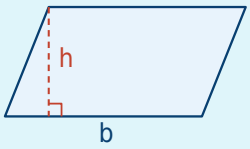
Parallelograms

Rule: area of a parallelogram

$A = b \times h$. A parallelogram can be cut and rearranged into a rectangle with the same base and height, which is why the formula is the same.

Example 3

Find the area of a parallelogram with base 12 cm and height 5 cm.



$A = 12 \times 5 = 60$ cm².

Watch out

Use the perpendicular height, not the slanted side. The slanted side is longer than the height.

Trapezia

Definition: trapezium

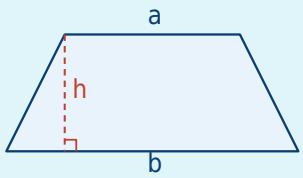
A **trapezium** is a quadrilateral with one pair of parallel sides.

Rule: area of a trapezium

$A = \frac{1}{2}(a + b)h$, where a and b are the parallel sides and h is the distance between them.

Example 4

Find the area of a trapezium with parallel sides 6 cm and 10 cm and height 4 cm.



$A = \frac{1}{2} \times (6 + 10) \times 4 = \frac{1}{2} \times 16 \times 4 = 32$ cm².

Compound shapes

Method: area of a compound shape

1. Split the shape into rectangles, triangles and other simple shapes (or subtract a cut-out from a larger shape).
2. Work out the missing lengths.
3. Find the area of each part and add (or subtract).
4. Write the units.

Example 5

An L-shape is made of a 10 cm by 4 cm rectangle and a 5 cm by 3 cm rectangle joined at the end. Find the total area.

$10 \times 4 = 40$ and $5 \times 3 = 15$. The total is $40 + 15 = 55$ cm².

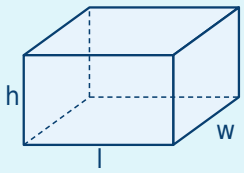
Volume of a cuboid

Rule: volume of a cuboid

$V = l \times w \times h$. Equivalently, volume is the area of the base multiplied by the height.

Example 6

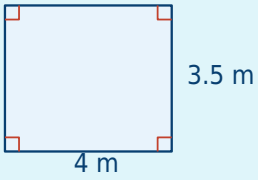
Find the volume of a cuboid 5 cm by 4 cm by 3 cm.



$V = 5 \times 4 \times 3 = 60$ cm³.

Example 7

A room is 4 m by 3.5 m. It is covered with square tiles of side 50 cm. How many tiles are needed?



Area of the room: $4 \times 3.5 = 14$ m². Area of one tile: $0.5 \times 0.5 = 0.25$ m². Number of tiles: $14 \div 0.25 = 56$.

Units

Key fact: converting area and volume units

$1\text{ m} = 100\text{ cm}$, so $1\text{ m}^2 = 10\,000\text{ cm}^2$ and $1\text{ m}^3 = 1\,000\,000\text{ cm}^3$. Also $1\text{ cm}^3 = 1\text{ ml}$ and $1000\text{ cm}^3 = 1$ litre.

Example 8

A tank is 50 cm by 40 cm by 30 cm. How many litres does it hold?

$50 \times 40 \times 30 = 60\,000$ cm³. Divide by 1 000: 60 litres.

Working backwards

Sometimes the area is given and you need a missing length. Rearrange the formula using inverse operations.

Example 9

A rectangle has area 54 cm² and width 6 cm. Find its length and perimeter.

$l = 54 \div 6 = 9$ cm. Perimeter $= 2 \times (9 + 6) = 30$ cm.

Example 10

A triangle has area 30 cm² and base 12 cm. Find its height.

$30 = \frac{1}{2} \times 12 \times h = 6h$, so $h = 5$ cm.

Example 11

A cuboid has a volume of 120 cm³, length 8 cm and width 5 cm. Find the height.

$8 \times 5 \times h = 120$, so $40h = 120$ and $h = 3$ cm.

Note

The surface area of a cuboid is the total area of its six faces: $2(lw + lh + wh)$. For a 5 by 4 by 3 cuboid it is $2(20 + 15 + 12) = 94$ cm².

Common mistakes

- Mixing units: convert everything to the same unit first.
- Using the slanted side as the height.
- Forgetting the $\frac{1}{2}$ in the triangle and trapezium formulae.
- Writing cm instead of cm² for area or cm³ for volume.

Watch out

Perimeter is a length (cm), area is in cm² and volume in cm³. Do not use the wrong unit.

Worked example in context

A rectangular garden is 12 m by 8 m. A path 1 m wide goes inside round all four edges. What is the area of the path?

The inner rectangle is $10 \times 6 = 60$ m². The whole garden is $12 \times 8 = 96$ m². The path area is $96 - 60 = 36$ m².

Common mistakes

- Using the sloping side instead of the perpendicular height of a triangle.
- Forgetting the half in the area of a triangle or trapezium.
- Mixing units: 1 m² is 10 000 cm², not 100 cm².

Did you know?

The Great Pyramid of Giza had a square base of about 230 m on each side, which is an area of about 53 000 m², larger than seven football pitches. Builders and farmers have used area and volume measurements for thousands of years to share land and to store grain.

Choosing the right formula

Perimeter, area and volume measure three very different things. Perimeter is a length, the distance all the way round a flat shape, so it is measured in cm or m. Area is the amount of flat surface inside a shape, measured in square units such as cm² or m². Volume is the amount of space inside a solid, measured in cubic units such as cm³ or m³. Before you start any calculation, decide which of the three the question is asking for: fencing and edging need a perimeter, carpet, paint and turf need an area, and water, sand and concrete need a volume.

Method: finding the area of a compound shape

1. Split the shape into rectangles, triangles or trapezia by drawing one or two extra lines.
2. Work out any missing lengths by adding or subtracting the lengths you are given.
3. Find the area of each part, writing the formula each time.
4. Add the areas together (or subtract if a piece has been cut out) and give the units.

Example 1

A garden is a rectangle 12 m by 8 m with a right-angled triangle 8 m long and 3 m high added to one end. Find its area.

We split the garden into the rectangle and the triangle. The rectangle has area $12 \times 8 = 96$ m². The triangle has base 8 m and perpendicular height 3 m, so its area is $\frac{1}{2} \times 8 \times 3 = 12$ m². The total area is $96 + 12 = 108$ m². Writing the two parts separately makes it easy for someone else to follow your reasoning and to spot any slip.

Example 2

A cuboid has a volume of 180 cm³. Its base is 6 cm by 5 cm. How tall is it?

The volume of a cuboid is length × width × height. The base has area $6 \times 5 = 30$ cm², so $30 \times h = 180$. Dividing both sides by 30 gives $h = 6$ cm. This “working backwards” idea is useful whenever you know the answer of a formula and need one of the measurements: write the formula, put in what you know, then use inverse operations.

Example 3

How many cubes of side 2 cm fit exactly inside a box 10 cm by 6 cm by 4 cm?

Along the length, $10 \div 2 = 5$ cubes fit. Along the width, $6 \div 2 = 3$ cubes fit, and up the height $4 \div 2 = 2$ cubes fit. Altogether $5 \times 3 \times 2 = 30$ cubes fit. We can check using volumes: the box has volume 240 cm³ and each cube has volume 8 cm³, and $240 \div 8 = 30$. Both methods agree.

Watch out

Area and volume conversions are not the same as length conversions. There are 100 cm in a metre, but there are $100 \times 100 = 10\,000$ cm² in a square metre and $100 \times 100 \times 100 = 1\,000\,000$ cm³ in a cubic metre. Drawing a 1 m square split into centimetre squares shows why: there are 100 rows of 100 small squares. The safest method is to convert all lengths to the same unit *before* you calculate.

Did you know?

The litre was defined in France in 1795 as the volume of a cube 10 cm on each side, which is exactly 1 000 cm³. That is why one millilitre is the same as one cubic centimetre, and why a cubic metre of water, a cube 1 m on each side, holds exactly 1 000 litres and weighs about one tonne. A bath holds roughly 150 litres.

Otto's tip

Sketch the shape and label every length. Then decide: is it perimeter (add the sides) or area (multiply)?