Lesson - Negative numbers, factors, multiples and primes

Year: Year 7

Numbers are the foundation of KS3 maths. In this chapter you will calculate with negative numbers, use factors, multiples and primes, and apply the priority of operations.

Starter activity

On a winter morning the temperature outside is 2 degrees below zero. By noon it is 5 degrees higher. By midnight it falls by 9 degrees. Draw a number line and show each change. What are the temperatures at noon and at midnight?

Negative numbers

Definition: negative number

A **negative number** is less than zero, such as -3. On a number line, numbers get smaller as you move left. We use negatives for temperatures, debts and heights below sea level.

Rule: adding and subtracting negatives

Adding a negative is the same as subtracting: $5 + (-3) = 5 - 3 = 2$. Subtracting a negative is the same as adding: $5 - (-3) = 5 + 3 = 8$.

Example 1

Work out $-4 + 9$, $-3 - 5$ and $2 - (-6)$.

$-4 + 9 = 5$ (move 9 right from -4). $-3 - 5 = -8$ (move 5 left). $2 - (-6) = 2 + 6 = 8$.

Example 2

The temperature at night is -7°C and it rises by 12 degrees by midday. Find the midday temperature.

$-7 + 12 = 5^\circ$C.

Multiplying and dividing negatives

Rule: signs when multiplying and dividing

Same signs give a positive answer. Different signs give a negative answer.

$(-3) \times (-4) = 12$, $(-3) \times 4 = -12$, $12 \div (-4) = -3$, $(-12) \div (-4) = 3$.

Example 3

Work out $(-6) \times 5$, $(-8) \div (-2)$ and $(-3) \times (-3) \times (-3)$.

$(-6) \times 5 = -30$. $(-8) \div (-2) = 4$. $(-3) \times (-3) = 9$ and $9 \times (-3) = -27$.

Watch out

-3^2 means $-(3^2) = -9$, but $(-3)^2 = 9$. Brackets matter when squaring a negative number.

Factors and multiples

Definition: factor and multiple

A **factor** of a number divides it exactly. A **multiple** of a number is in its times table. The factors of 12 are 1, 2, 3, 4, 6 and 12. The multiples of 12 are 12, 24, 36 and so on.

Method: finding all the factors of a number

1. Find pairs of numbers that multiply to give the number, starting with $1 \times n$.
2. Test 2, 3, 4 and so on in order.
3. Stop when the pairs repeat, then list the factors in order.

Example 4

List all the factors of 48.

The pairs are 1×48, 2×24, 3×16, 4×12 and 6×8. The factors are 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

Prime numbers

Definition: prime number

A **prime number** has exactly two factors, 1 and itself. The primes up to 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. The number 1 is not prime. A number with more than two factors is **composite**.

Example 5

Is 91 a prime number?

Test small primes: $91 = 7 \times 13$. It has other factors, so 91 is not prime.

Prime factorisation

Definition: prime factorisation

Every whole number greater than 1 is either prime or can be written as a product of prime factors in just one way. Writing it like this is **prime factorisation**. A factor tree helps.

Method: writing a number as a product of primes

1. Split the number into any two factors.
2. Keep splitting each factor until all are prime.
3. Write the primes in order, using powers for repeats.

Example 6

Write 72 as a product of prime factors.

$72 = 8 \times 9 = 2 \times 2 \times 2 \times 3 \times 3$. So $72 = 2^3 \times 3^2$.

HCF and LCM

Definition: HCF and LCM

The **highest common factor (HCF)** of two numbers is the biggest number that divides into both. The **lowest common multiple (LCM)** is the smallest number that both divide into.

Example 7

Find the HCF and LCM of 12 and 18.

$12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. HCF: the lowest powers shared, $2 \times 3 = 6$. LCM: the highest powers, $2^2 \times 3^2 = 36$.

Example 8

Two buses leave a station together. One returns every 12 minutes and the other every 18 minutes. When do they next leave together?

The LCM of 12 and 18 is 36, so after 36 minutes.

Squares, cubes and roots

Key fact: squares, cubes and roots

Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144. Cubes: 1, 8, 27, 64, 125, 1000. The square root reverses squaring: $\sqrt{49} = 7$. The cube root reverses cubing: $\sqrt[3]{64} = 4$.

Example 9

Work out $\sqrt{81} + 4^3$.

$9 + 64 = 73$.

Priority of operations (BIDMAS)

Rule: BIDMAS

Work in this order: Brackets, Indices, Division and Multiplication, then Addition and Subtraction. Do division and multiplication left to right, and addition and subtraction left to right.

Example 10

Work out $8 + 3 \times (7 - 2)^2 \div 5$.

Brackets: $7 - 2 = 5$. Indices: $5^2 = 25$. Then $3 \times 25 = 75$ and $75 \div 5 = 15$. Finally $8 + 15 = 23$.

Common mistakes

- Thinking 1 is a prime number.
- Thinking all odd numbers are prime (9 and 15 are not).
- Treating -3^2 as 9.
- Confusing HCF and LCM: the HCF is never bigger than the numbers, the LCM never smaller.

Watch out

The only even prime is 2. Do not confuse factors (which divide into the number) with multiples (which the number divides into).

Worked example in context

A diver is 12 m below the surface. She swims up 5 m and then dives down 9 m. What is her depth now?

Start at -12. Swim up 5: $-12 + 5 = -7$. Dive down 9: $-7 - 9 = -16$. She is 16 m below the surface.

Common mistakes

- Thinking that -9 is greater than -2 because 9 is greater than 2.
- Treating "minus a negative" as another minus.
- Confusing HCF (the greatest factor shared) with LCM (the smallest multiple shared).

Did you know?

Prime numbers are the building blocks of all whole numbers. In 2018 a computer found a prime number with over 23 million digits. Prime numbers also protect your data online: bank cards and websites use huge primes to keep secrets safe.

Starter activity: the number line game

Draw a number line from -10 to 10 and put a counter on zero. Roll a dice twice: the first roll tells you how far to move, and a coin decides the direction (heads to the right, tails to the left). After five turns, write down a calculation that describes all your moves, such as $0 + 4 - 6 - 3 + 5 - 2 = -2$. This simple game shows that adding moves you to the right, subtracting moves you to the left, and that the final position can be below zero even when you started at zero.

Method: subtracting a negative number

1. Rewrite the calculation so that “minus a negative” becomes “plus a positive”: $a - (-b) = a + b$.
2. Start at the first number on a number line.
3. Move to the right for addition and to the left for subtraction.
4. Read off where you land, and check the size of the answer makes sense.

Example 1

The temperature at midnight is -3 °C. By midday it is 8 °C. By how many degrees has the temperature risen?

The rise is the difference between the two temperatures, which is the later value minus the earlier value: $8 - (-3)$. Subtracting a negative is the same as adding, so this is $8 + 3 = 11$. The temperature has risen by 11 degrees. You can also count on a number line: 3 steps from -3 up to 0, then 8 steps from 0 up to 8, which is 11 steps altogether.

Example 2

Find the highest common factor and the lowest common multiple of 18 and 24 using prime factors.

First write each number as a product of primes: $18 = 2 \times 3^2$ and $24 = 2^3 \times 3$. For the HCF we take each prime that appears in *both* numbers, using the lower power: $2^1 \times 3^1 = 6$. For the LCM we take every prime that appears in *either* number, using the higher power: $2^3 \times 3^2 = 72$. A useful check is that HCF × LCM equals the product of the two numbers: $6 \times 72 = 432$ and $18 \times 24 = 432$.

Example 3

Is 119 a prime number?

We only need to test prime divisors up to the square root of 119, which is just under 11, so we try 2, 3, 5 and 7. 119 is odd, so 2 does not divide it. Its digits add to 11, which is not a multiple of 3, so 3 does not divide it. It does not end in 0 or 5, so 5 does not divide it. But $7 \times 17 = 119$, so 119 is **not** prime. Testing only up to the square root saves a lot of time.

Watch out

Be careful with powers of negative numbers. $(-3)^2 = (-3) \times (-3) = 9$, but -3^2 means $-(3^2) = -9$, because the power is worked out before the minus sign is applied. Calculators follow this rule too, so always type the brackets when you want to square a negative number.

Did you know?

Negative numbers were accepted surprisingly late. Chinese mathematicians used red and black counting rods for positive and negative numbers more than 2 000 years ago, and the Indian mathematician Brahmagupta wrote rules for them in the year 628, describing them as “debts”. Yet many European mathematicians still called them “absurd numbers” as late as the 1700s.

Otto's tip

Use a factor tree and keep splitting until every branch ends in a prime. Circle the primes as you go!