Lesson - Fractions

Year: Year 7

Fractions describe parts of a whole. In this chapter you will simplify, compare and calculate with fractions and mixed numbers, using all four operations.

Starter activity

A pizza is cut into 8 equal slices. Ben eats 3 slices and Chloe eats 2 slices. What fraction of the pizza has been eaten altogether? What fraction is left? Write each answer as a fraction, then try to explain why the denominator stays the same when you add.

What is a fraction?

Definition: fraction

A **fraction** $\frac{a}{b}$ represents a equal parts of a whole divided into b equal parts. The top number is the **numerator** and the bottom number is the **denominator**.

Definition: types of fraction

A **proper fraction** has a numerator smaller than the denominator, such as $\frac{3}{4}$. An **improper fraction** has a numerator bigger than or equal to the denominator, such as $\frac{7}{4}$. A **mixed number** has a whole number and a fraction, such as $1\frac{3}{4}$.

Equivalent fractions and simplifying

Rule: equivalent fractions

Multiplying or dividing the numerator and denominator by the same number gives an equivalent fraction. To simplify, divide both by their highest common factor (HCF).

Example 1

Simplify $\frac{24}{36}$.

The HCF of 24 and 36 is 12. $\frac{24 \div 12}{36 \div 12} = \frac{2}{3}$.

Mixed numbers and improper fractions

Method: converting between mixed numbers and improper fractions

1. Mixed to improper: multiply the whole number by the denominator, add the numerator, keep the denominator.
2. Improper to mixed: divide the numerator by the denominator; the quotient is the whole number and the remainder is the new numerator.

Example 2

Write $3\frac{2}{5}$ as an improper fraction and $\frac{23}{6}$ as a mixed number.

$3 \times 5 + 2 = 17$, so $\frac{17}{5}$. $23 \div 6 = 3$ remainder 5, so $3\frac{5}{6}$.

Comparing fractions

Rule: comparing fractions

Write the fractions with a common denominator, then compare the numerators.

Example 3

Which is larger, $\frac{5}{6}$ or $\frac{7}{9}$?

The common denominator is 18. $\frac{15}{18}$ and $\frac{14}{18}$. So $\frac{5}{6}$ is larger.

Adding and subtracting fractions

Method: adding and subtracting fractions

1. Find a common denominator (the LCM of the denominators).
2. Write each fraction as an equivalent fraction with that denominator.
3. Add or subtract the numerators and keep the denominator.
4. Simplify.

Example 4

Work out $\frac{3}{4} + \frac{2}{3}$.

The common denominator is 12. $\frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$.

Example 5

Work out $3\frac{1}{2} - 1\frac{3}{4}$.

As improper fractions: $\frac{7}{2} - \frac{7}{4} = \frac{14}{4} - \frac{7}{4} = \frac{7}{4} = 1\frac{3}{4}$.

Watch out

Never add the denominators. $\frac{1}{2} + \frac{1}{3}$ is $\frac{5}{6}$, not $\frac{2}{5}$.

Fraction of an amount

Rule: fraction of an amount

To find $\frac{a}{b}$ of an amount, divide by b and multiply by a.

Example 6

Find $\frac{3}{8}$ of 56.

$56 \div 8 = 7$ and $7 \times 3 = 21$.

Multiplying fractions

Rule: multiplying fractions

Multiply the numerators and multiply the denominators. Cancel common factors first to make the numbers smaller. Write mixed numbers as improper fractions.

Example 7

Work out $\frac{4}{9} \times \frac{3}{8}$.

Cancel: 4 and 8 share 4, and 3 and 9 share 3. This gives $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

Example 8

Work out $1\frac{1}{2} \times 2\frac{2}{3}$.

$\frac{3}{2} \times \frac{8}{3} = \frac{24}{6} = 4$.

Dividing fractions

Rule: dividing fractions

To divide by a fraction, turn the second fraction upside down (find its reciprocal) and multiply: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

Example 9

Work out $\frac{3}{4} \div \frac{3}{8}$.

$\frac{3}{4} \times \frac{8}{3} = \frac{24}{12} = 2$.

More worked examples

Example 10

Sam eats $\frac{1}{4}$ of a pizza and Jo eats $\frac{1}{3}$. What fraction is left?

$\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$, so $1 - \frac{7}{12} = \frac{5}{12}$ is left.

Example 11

A jug holds $2\frac{1}{2}$ litres. How many $\frac{1}{4}$-litre glasses can be filled?

$\frac{5}{2} \div \frac{1}{4} = \frac{5}{2} \times 4 = 10$ glasses.

Fractions in context

Fractions appear in recipes, money and measurement. When a question has several steps, work out what fraction of the whole each part is, and check that the parts add up to 1.

Example 12

A class has 30 pupils. $\frac{2}{5}$ walk to school, $\frac{1}{3}$ come by bus and the rest are driven. How many are driven?

Walk: $\frac{2}{5} \times 30 = 12$. Bus: $\frac{1}{3} \times 30 = 10$. Driven: $30 - 12 - 10 = 8$ pupils.

Example 13

Work out $\frac{2}{3}$ of $\frac{3}{4}$ of 24.

$\frac{3}{4}$ of 24 is 18, and $\frac{2}{3}$ of 18 is 12.

Example 14

Order $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{7}{10}$ from smallest to largest.

Use a common denominator of 40: $\frac{30}{40}$, $\frac{25}{40}$, $\frac{28}{40}$. The order is $\frac{5}{8}$, $\frac{7}{10}$, $\frac{3}{4}$.

Note

A whole number can be written as a fraction over 1. So $5 = \frac{5}{1}$, and $4 \div \frac{1}{2} = \frac{4}{1} \times \frac{2}{1} = 8$: there are eight halves in 4.

Common mistakes

- Adding the numerators and the denominators.
- Forgetting to turn the second fraction upside down when dividing.
- Using a common denominator when multiplying (it is not needed).
- Not simplifying the answer.

Worked example in context

A recipe needs $\frac{3}{4}$ of a cup of flour. Lucy wants to make half the recipe. How much flour does she need?

Half of $\frac{3}{4}$ means $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$. Lucy needs $\frac{3}{8}$ of a cup.

Common mistakes

- Adding the denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.
- Flipping the wrong fraction when dividing: only the second fraction is flipped.
- Multiplying mixed numbers without changing them to improper fractions first.

Did you know?

The word *fraction* comes from the Latin *fractio*, meaning "breaking". The ancient Egyptians wrote almost every fraction as a sum of unit fractions such as $\frac{1}{2} + \frac{1}{4}$. Musicians use fractions all the time: a quaver is half the length of a crotchet.

Starter activity: sharing chocolate bars

Imagine three friends sharing two identical chocolate bars equally. Each bar can be broken into three equal pieces, giving six pieces altogether, so each friend gets two pieces, which is $\frac{2}{3}$ of a bar. This shows an important idea: a fraction is also a division. $\frac{2}{3}$ means $2 \div 3$. Thinking of fractions as divisions helps you to convert them into decimals, to find fractions of amounts and to understand why an improper fraction such as $\frac{7}{2}$ is the same as $3\frac{1}{2}$.

Method: choosing the right operation

1. Adding or subtracting: find a common denominator first, then add or subtract the numerators only.
2. Multiplying: change mixed numbers into improper fractions, cancel any common factors, then multiply the numerators and multiply the denominators.
3. Dividing: keep the first fraction, change the division into a multiplication and flip the second fraction.
4. Finish by simplifying and, if the question uses mixed numbers, convert an improper answer back into a mixed number.

Example 1

Work out $3\frac{2}{3} - 1\frac{3}{4}$.

One way is to change both numbers into improper fractions. $3\frac{2}{3} = \frac{11}{3}$ and $1\frac{3}{4} = \frac{7}{4}$. The lowest common multiple of 3 and 4 is 12, so we write $\frac{44}{12} - \frac{21}{12} = \frac{23}{12}$. As a mixed number this is $1\frac{11}{12}$. A sensible check: $3\frac{2}{3}$ is a little less than 4 and $1\frac{3}{4}$ is a little less than 2, so the answer should be close to 2, and $1\frac{11}{12}$ is.

Example 2

Work out $\frac{8}{15} \times \frac{5}{12}$, cancelling first.

Before multiplying, we look for a numerator and a denominator that share a factor. 8 and 12 share the factor 4, so they become 2 and 3. 5 and 15 share the factor 5, so they become 1 and 3. The calculation is now $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$. Cancelling first keeps the numbers small, and the answer is already in its simplest form.

Example 3

A ribbon is $2\frac{1}{4}$ m long. How many pieces of length $\frac{3}{8}$ m can be cut from it?

"How many pieces fit" is a division question. $2\frac{1}{4} = \frac{9}{4}$, so we work out $\frac{9}{4} \div \frac{3}{8} = \frac{9}{4} \times \frac{8}{3} = \frac{72}{12} = 6$. Exactly six pieces can be cut. Notice that dividing by a fraction smaller than 1 gave a bigger answer, which makes sense because each piece is short.

Example 4

Which is bigger, $\frac{5}{8}$ or $\frac{7}{11}$?

The denominators 8 and 11 have no common factor, so the lowest common denominator is $8 \times 11 = 88$. We write $\frac{5}{8} = \frac{55}{88}$ and $\frac{7}{11} = \frac{56}{88}$. Now the pieces are the same size, and 56 pieces are more than 55 pieces, so $\frac{7}{11}$ is bigger, but only just. Another way is to change both fractions into decimals: $5 \div 8 = 0.625$ and $7 \div 11 = 0.636\ldots$, which leads to the same conclusion. Choose whichever method gives easier numbers. When the fractions are very close, as here, write your working carefully, because a guess based on how the fractions look is very likely to be wrong. Remember too that with the same numerator, the fraction with the smaller denominator is bigger, since the whole is cut into fewer, larger pieces.

Watch out

When you find a fraction of an amount, divide by the denominator *before* multiplying by the numerator. To find $\frac{3}{5}$ of 40, work out $40 \div 5 = 8$ and then $8 \times 3 = 24$. Doing it the other way round gives the same answer but with larger numbers, and dividing by the numerator instead of the denominator is a very common mistake.

Did you know?

The ancient Egyptians wrote almost every fraction as a sum of different unit fractions, which are fractions with a numerator of 1. For example, they wrote $\frac{3}{4}$ as $\frac{1}{2} + \frac{1}{4}$. The Rhind Papyrus, a mathematics scroll more than 3 500 years old, contains a long table of these unit fraction sums, used by scribes for sharing bread and beer fairly between workers. Today we know that every fraction can be written in this way, often in several different ways.

Otto's tip

Before you start, ask: Is this add or subtract (common denominator!), or multiply or divide (no common denominator needed)? Choose the right tool!