Lesson - Expanding brackets and substitution

Year: Year 7

Brackets group terms together. In this chapter you will expand a single bracket, substitute into formulae and form simple expressions.

Starter activity

A baker sells cupcakes at £2 each and cookies at £1.50 each. Write a formula for the cost, C, of x cupcakes and y cookies. Use it to find the cost of 5 cupcakes and 8 cookies. Why does a formula save time compared with working each bill out from scratch?

What does a bracket mean?

Definition: bracket

A **bracket** groups terms. $3(x+2)$ means 3 lots of $(x+2)$, which is $x+2+x+2+x+2=3x+6$.

Expanding a single bracket

Rule: expanding a bracket

Multiply every term inside the bracket by the term outside: $a(b+c)=ab+ac$ and $a(b-c)=ab-ac$.

Method: expanding a single bracket

1. Multiply the term outside by the first term inside.
2. Multiply the term outside by the second term inside.
3. Keep the sign between the terms.

Example 1

Expand $4(x+5)$.

$4\times x+4\times 5=4x+20$.

Example 2

Expand $3x(2x-7)$.

$3x \times 2x = 6x^2$ and $3x \times 7 = 21x$. The answer is $6x^2 - 21x$.

Negative numbers outside the bracket

Rule: negative multipliers

A negative number outside the bracket changes the sign of every term inside: $-2(x - 3) = -2x + 6$.

Example 3

Expand $-5(2a - 4)$.

$-5 \times 2a = -10a$ and $-5 \times (-4) = +20$. The answer is $-10a + 20$.

Watch out

Multiply every term, not just the first. $3(x + 2)$ is $3x + 6$, not $3x + 2$. And be careful with minus signs.

Expand and simplify

Method: expanding and simplifying

1. Expand each bracket.
2. Collect like terms.
3. Write the answer in a tidy order.

Example 4

Expand and simplify $3(x + 4) + 2(x - 1)$.

$3x + 12 + 2x - 2 = 5x + 10$.

Example 5

Expand and simplify $5(2y + 1) - 3(y + 4)$.

$10y + 5 - 3y - 12 = 7y - 7$.

Substitution into formulae

Definition: formula

A **formula** is a rule that links quantities using letters. For example, the area of a

triangle is $A = \frac{1}{2}bh$, and the speed formula is $v = u + at$.

Example 6

Use $v = u + at$ to find v when $u = 5$, $a = 2$ and $t = 3$.

$v = 5 + 2 \times 3 = 5 + 6 = 11$.

Example 7

Use $C = \frac{5}{9}(F - 32)$ to convert $F = 68$ to Celsius.

$C = \frac{5}{9} \times (68 - 32) = \frac{5}{9} \times 36 = 20^{\circ}\text{C}$.

Note

Always follow BIDMAS: work out brackets first, then indices, then multiplication and division, then addition and subtraction.

Forming expressions

Example 8

A number is n. Write expressions for: 5 more than the number, 3 times the number, and twice the number plus 7, all multiplied by 4.

$n + 5$, $3n$ and $4(2n + 7)$, which expands to $8n + 28$.

Example 9

A rectangle has width w and length $w + 4$. Write and expand an expression for its area.

Area $= w(w + 4) = w^2 + 4w$.

Why expanding works

Think of a rectangle with width 3 and length $x + 2$. It can be split into a rectangle $3 \times x$ and a rectangle 3×2. The total area is $3x + 6$, which is the same as $3(x + 2)$. The expanded form has no brackets and the bracket form is the **factorised** form. Both are equivalent: they give the same value for every x.

Example 10

Show that $2(x + 5)$ and $2x + 10$ are equal when $x = 4$.

$2(4+5) = 2 \times 9 = 18$ and $2 \times 4 + 10 = 18$. They are equal.

Example 11

Expand and simplify $2(3x+1) - 4(x-2)$.

$6x + 2 - 4x + 8$. Careful with the sign: $-4 \times (-2) = +8$. Collect like terms: $2x + 10$.

Example 12

Use $s = ut + \frac{1}{2}at^2$ to find s when $u = 3$, $t = 4$ and $a = 2$.

$s = 3 \times 4 + \frac{1}{2} \times 2 \times 4^2 = 12 + 16 = 28$. Indices come before multiplication.

Note

To check an expansion, substitute a simple number such as $x = 1$ or $x = 2$ into both the original and the expanded expression. If they differ, you have made a mistake.

Common mistakes

- Only multiplying the first term.
- Forgetting that a negative times a negative is positive.
- Treating $3(x+2)$ as $3x + 2$.
- Substituting without brackets when the value is negative.

Watch out

When substituting a negative number, use brackets. If $x = -3$, then $x^2 = (-3)^2 = 9$, not -9.

Worked example in context

The cost of hiring a canoe is £6 plus £2.50 per hour. Write a formula and find the cost for 4 hours.

The formula is $C = 6 + 2.5h$. For $h = 4$: $C = 6 + 10 = 16$. The cost is £16.

Common mistakes

- Multiplying only the first term in a bracket: $3(x+5) \neq 3x + 5$.
- Forgetting to change the sign when expanding a bracket with a negative number in front.
- Replacing x^2 with $x \times 2$ instead of x times x.

Did you know?

Formulae are everywhere: the formula for converting temperatures, for working out how long a journey takes, or for the speed of a falling object. Engineers use formulae to design bridges and rockets before building anything.

Starter activity: the area model

Before we look at more examples, try this short activity. Draw a rectangle and split it into two smaller rectangles with a vertical line. Label the height of the whole rectangle 3, the width of the left part x and the width of the right part 5. The area of the whole rectangle can be written in two ways. As one big rectangle it is height times total width, which is $3(x+5)$. As two small rectangles it is $3x$ plus 15. Both describe exactly the same area, so $3(x+5) = 3x + 15$. This picture is the reason why expanding brackets works, and it helps you remember that the number outside must multiply *every* term inside.

Method: the grid method for expanding

1. Draw a table with the term outside the bracket at the start of the row.
2. Write each term inside the bracket, with its sign, at the top of a column.
3. Multiply the row term by each column term and write the results in the boxes.
4. Add the boxes together and collect any like terms to give the expanded expression.

Example 1

Expand $4x(3x - 2y + 5)$.

The term outside the bracket is $4x$ and there are three terms inside, so we need three multiplications. First, $4x \times 3x = 12x^2$, because $4 \times 3 = 12$ and $x \times x = x^2$. Second, $4x \times (-2y) = -8xy$; a positive times a negative gives a negative. Third, $4x \times 5 = 20x$. Putting the three results together gives $12x^2 - 8xy + 20x$. None of these are like terms, so the expression cannot be simplified any further.

Example 2

Expand and simplify $5(2x + 3) - 2(4x - 1)$.

We expand each bracket separately and keep track of the signs. The first bracket gives $10x + 15$. For the second bracket the number outside is -2, so $-2 \times 4x = -8x$ and $-2 \times (-1) = +2$. Now we write everything in one line: $10x + 15 - 8x + 2$. Collecting the x terms gives $10x - 8x = 2x$, and collecting the numbers gives $15 + 2 = 17$. The answer is $2x + 17$. Checking with $x = 1$: the original is $5 \times 5 - 2 \times 3 = 25 - 6 = 19$, and $2 + 17 = 19$, so the answer is correct.

Example 3

Use the formula $A = \frac{1}{2}(a+b)h$ to find the area of a trapezium with parallel sides 6 cm and 10 cm and height 4 cm.

We replace each letter by its value: $a = 6$, $b = 10$ and $h = 4$. Brackets come first in the order of operations, so $a + b = 16$. Then we multiply: $\frac{1}{2} \times 16 \times 4 = 32$. The area is 32 cm². Writing the substituted formula before calculating, $A = \frac{1}{2}(6+10) \times 4$, makes your working easy to follow and earns method marks even if you slip later.

Factorising: expanding in reverse

Factorising is the opposite of expanding. Instead of multiplying out a bracket, we look for the highest common factor of all the terms and put it outside a bracket. For example, in $6x + 15$ both terms are multiples of 3, so $6x + 15 = 3(2x + 5)$. To check a factorisation, simply expand it again and see whether you get back to the expression you started with. If the terms inside the bracket still have a common factor, you have not factorised *fully*, so look again.

Example 4

Factorise fully $12x^2 - 18x$.

We look at the numbers and the letters separately. The highest common factor of 12 and 18 is 6. Both terms also contain at least one x, so x is a common factor too. Together the highest common factor is $6x$. Dividing each term by $6x$ gives $12x^2 \div 6x = 2x$ and $-18x \div 6x = -3$. So $12x^2 - 18x = 6x(2x - 3)$. If you had only taken out 6, giving $6(2x^2 - 3x)$, the expression would be factorised but not *fully*, because $2x^2 - 3x$ still has the common factor x.

Method: factorising fully

1. Find the highest common factor of the numbers.
2. Find the letters that appear in every term, using the lowest power of each.
3. Write the highest common factor outside a bracket and divide each term by it to find what goes inside.
4. Expand your answer to check that you get back the original expression.

Watch out

A minus sign in front of a bracket multiplies every term inside by -1. So $-(2x - 7)$ is $-2x + 7$, not $-2x - 7$. Many pupils change the sign of the first term and forget the second. Writing the hidden -1 in front of the bracket, as $-1(2x - 7)$, helps you to remember. Then check the signs one more time before you collect like terms.

Did you know?

Mental arithmetic tricks often hide an expanded bracket. To work out 7×98, think of it as $7(100 - 2) = 700 - 14 = 686$. Shopkeepers in the past used exactly this kind of trick to add up prices quickly, long before calculators existed. Try it yourself: $6 \times 103 = 6(100 + 3) = 600 + 18 = 618$, and $9 \times 49 = 9(50 - 1) = 450 - 9 = 441$. With a little practice you can multiply many awkward numbers in your head by splitting one of them into a round number plus or minus a small one.

Otto's tip

Draw arrows from the outside term to each term inside the bracket. Every term gets a visit!