Answers - Expanding brackets and substitution

Year: Year 7

Answer to Question 1

Method: replace each letter by its value, putting negative numbers in brackets, then follow the order of operations (brackets, powers, multiplication and division, addition and subtraction).

a. Replace x by 4: $3 \times 4 + 7 = 12 + 7 =$**19**.
b. $2 \times 4 - 5 \times (-2) = 8 - (-10) = 8 + 10 =$**18**. Subtracting a negative is adding.
c. Square first: $4^2 = 16$, then $16 + 5 =$**21**.
d. $xy = 4 \times (-2) = -8$, then $-8 + 5 = -3$.
e. $\frac{3 \times 4}{6} = \frac{12}{6} = 2$, then $2 - (-2) = 2 + 2 =$**4**.
f. Bracket first: $4 + (-2) = 2$, then $2 \times 2 =$**4**.

Answer to Question 2

To expand, multiply the term outside the bracket by every term inside, taking care with the signs: same signs give a positive result, different signs give a negative result.

a. Multiply each term by 3: $3 \times x = 3x$ and $3 \times 4 = 12$. Answer $3x + 12$.
b. $5 \times 2a = 10a$ and $5 \times (-3) = -15$: $10a - 15$.
c. $-2 \times y = -2y$ and $-2 \times 6 = -12$: $-2y - 12$.
d. $x \times x = x^2$ and $x \times 7 = 7x$: $x^2 + 7x$.
e. $4a \times 2a = 8a^2$ and $4a \times (-b) = -4ab$: $8a^2 - 4ab$.
f. $-3 \times 2m = -6m$ and $-3 \times (-5) = +15$: $-6m + 15$.

Answer to Question 3

Expand each bracket separately, write all the terms in one line, then collect the like terms (x terms together, numbers together).

a. Expand: $3x + 6 + 4x + 4$. Collect: $7x + 10$. Answer $7x + 10$.
b. $5a + 15 + 2a - 8$. The a terms make $7a$, the numbers $15 - 8 = 7$: $7a + 7$.
c. $6y - 2 - 3y - 6$; the -3 multiplies both terms. Collect: $3y - 8$.
d. $-(p - 5) = -p + 5$, so $8p + 4 - p + 5 = 7p + 9$.
e. $x^2 + 3x + 2x + 6$. Collect the x terms: $x^2 + 5x + 6$.

Answer to Question 4

a. The HCF of 6 and 9 is 3: $3(2x+3)$. Check: $3\times 2x+3\times 3=6x+9$.
b. HCF of 10 and 15 is 5: $5(2a-3)$.
c. Both terms contain 4 and y: $4y(y+2)$. Check: $4y^2+8y$.
d. HCF of 12ab and 18b is 6b: $6b(2a+3)$.
e. Both terms contain x: $x(x-5)$.
f. Numbers: HCF 5. Letters: both have m and n. Take out 5mn: $5mn(3m+2n)$.

Answer to Question 5

In each case write the formula, replace each letter by its value, and then calculate using the order of operations.

a. Substitute: $v=5+3\times 4=5+12=$**17**. Multiplication before addition.
b. $A=\frac{1}{2}\times 8\times 5=4\times 5=$**20**.
c. Bracket first: $7.5+4=11.5$, then $P=2\times 11.5=$**23**.
d. $ut=2\times 3=6$ and $\frac{1}{2}at^2=\frac{1}{2}\times 4\times 9=18$. So $s=6+18=$**24**.

Writing the substituted formula before calculating shows your method clearly and earns marks even if a later step goes wrong.

Answer to Question 6

Put $b=-4$ in brackets every time. Squaring a negative number gives a positive answer, and subtracting a negative number is the same as adding.

a. $b^2=(-4)^2=16$, so $2\times 3+16=6+16=$**22**.
b. Bracket first: $6+(-4)=2$, then $2^2=$**4**.
c. $ab=3\times(-4)=-12$, then $-12-(-4)=-12+4=-8$.
d. $9-16=-7$. Note $b^2=16$ is positive.
e. $a-b=3-(-4)=7$, then $3\times 7=$**21**.

Answer to Question 7

a. Substitute $m=200$: $C=15+0.04\times 200=15+8=$**£23**.
b. With $m=0$, $C=15+0=$**£15**. This is the fixed monthly charge, paid even if no extra minutes are used.
c. $15+0.04m=27$. Subtract 15: $0.04m=12$. Divide by 0.04: $m=300$. She used **300 minutes**.
d. $15+0.04\times 350=15+14=$**£29**. This is less than £30, so her mum is **not** right.

Answer to Question 8

a. $C=3.2+1.8\times 5=3.2+9=$**£12.20**.

b. $C = 3.2 + 1.8 \times 12.5 = 3.2 + 22.5 =$**£25.70**.
c. $3.2 + 1.8m = 21.2$. Subtract 3.2: $1.8m = 18$. Divide by 1.8: $m =$**10 miles**.
d. Second firm: $C = 2 + 2m$. For 10 miles: $2 + 20 = £22$. Leeds: $3.2 + 18 = £21.20$. The **Leeds firm** is cheaper by 80p.

Each time we substitute into the same formula or rearrange it by doing the inverse operations in reverse order: first subtract the fixed charge, then divide by the price per mile.

Answer to Question 9

The path is the part of the large rectangle that is not lawn, so its area is the difference between the two areas.

a. Area = length × width: $x(x - 2) = x^2 - 2x$ m².
b. $x(x + 2) = x^2 + 2x$ m².
c. Path = large rectangle − lawn: $(x^2 + 2x) - (x^2 - 2x) = x^2 + 2x - x^2 + 2x = 4x$ m².
d. Substitute $x = 8$: $4 \times 8 =$**32 m²**. Check: $80 - 48 = 32$.

Answer to Question 10

a. Multiply both terms by 30: $30x + 30 \times 5 = 30x + 150$.
b. $30 \times 12 + 150 = 360 + 150 =$**£510**.
c. $30x + 150 = 630$. Subtract 150: $30x = 480$. Divide by 30: $x =$**16**, so the trip costs £16 each.
d. $30 \times 12 + 5 = 365$, compared with £510: a difference of **£145**. It is wrong because the bracket shows that every pupil pays the £5, not just one.

Answer to Question 11

a. Add 4: $n + 4$. Times 3: $3(n + 4) = 3n + 12$. Subtract 12: $3n$. Divide by 3: n.
b. $9 + 4 = 13$, $13 \times 3 = 39$, $39 - 12 = 27$, $27 \div 3 = 9$. You get back **9**.
c. The expression simplifies to n for every value of n, so the game always returns the starting number.
d. For example: add 3, multiply by 2, subtract 6. $2(n + 3) - 6 = 2n + 6 - 6 = 2n$, double the starting number.

Answer to Question 12

a. $F = 1.8 \times 180 + 32 = 324 + 32 =$**356 °F**.
b. 20 °C: $1.8 \times 20 + 32 = 36 + 32 =$**68 °F**. 100 °C: $180 + 32 =$**212 °F**, the boiling point of water.
c. $1.8C + 32 = 392$. Subtract 32: $1.8C = 360$. Divide by 1.8: $C =$**200 °C**.

d. $1.8 \times (-40) + 32 = -72 + 32 = -40$. So -40 is the same in both scales.

To go from Fahrenheit back to Celsius we undo the formula: subtract 32 first, then divide by 1.8.

Answer to Question 13

a. Substitute into $4n + 3$: tower 4 uses $16 + 3 =$**19**, tower 10 uses $40 + 3 =$**43**.
b. $4 \times 25 + 3 = 100 + 3 =$**103** blocks.
c. $4n + 3 = 83$, so $4n = 80$ and $n = 20$. It is tower **20**.
d. $4 \times 6 + 3 = 24 + 3 = 27$. Tower 6 uses 27 blocks, so Alfie is **wrong**. Also 28 is not 3 more than a multiple of 4.

Answer to Question 14

a. Expand: $4 \times a + 4 \times c - 5 = 4a + 4c - 5$.
b. $2 \times 9 + 3 \times 6 = 18 + 18 =$**£36**.
c. $4 \times 9 + 4 \times 6 - 5 = 36 + 24 - 5 =$**£55**.
d. Without the deal: $4 \times 9 + 4 \times 6 = £60$. The deal costs £55, so it saves **£5**, the -5 in the formula.

The group deal costs £55 compared with £60 for the same eight tickets, so it is worth buying.

Answer to Question 15

Remember that the perimeter is the distance all the way round (add the four sides), while the area is length multiplied by width.

a. $2(2x + 3) + 2(x + 1) = 4x + 6 + 2x + 2 = 6x + 8$ cm.
b. Area = length × width: $(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3$ cm².
c. With $x = 4$: perimeter $24 + 8 =$**32 cm**; area $32 + 20 + 3 =$**55 cm²** (check: $11 \times 5 = 55$).
d. $6x + 8 = 50$, so $6x = 42$ and $x = 7$. Length $2 \times 7 + 3 = 17$, width $7 + 1 = 8$: **17 cm by 8 cm**.

Answer to Question 16

a. A: $C = 8 + 0.1m$. B: $C = 2 + 0.15m$.
b. Substitute 40, 80, 120, 160. A: 12, 16, 20, 24. B: 8, 14, 20, 26.
c. $8 + 0.1m = 2 + 0.15m$. Subtract $0.1m$ and 2: $6 = 0.05m$, so $m =$**120 minutes** (both cost £20).
d. At 100 minutes A costs $8 + 10 = £18$ and B costs $2 + 15 = £17$. **Plan B** is cheaper by **£1**.

Answer to Question 17

a. Ali only multiplied the x by 3. The 5 must be multiplied too: $3x + 15$.
b. Beth forgot that $-2 \times (-4) = +8$: $-2x + 8$.
c. Carl squared each term. $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$.
d. Dina lost the brackets. $(-3)^2 = (-3) \times (-3) =$**9**, because a negative times a negative is positive.

A quick check is to substitute a number such as $x = 1$ into both sides: if the two sides give different values, the expansion is wrong.

Answer to Question 18

a. Perimeter: $2x + 2(x + 10) = 2x + 2x + 20 = 4x + 20$ metres.
b. Fence cost $6.5(4x + 20) = 26x + 130$, plus the £75 gate: $T = 26x + 205$.
c. $26 \times 40 + 205 = 1040 + 205 =$**£1 245**.
d. $26x + 205 \leq 1000$, so $26x \leq 795$ and $x \leq 30.58$. The greatest whole value is $x = 30$ (cost £985).

The answer to an inequality like this must be rounded down, because rounding up to 31 would make the cost £1 011, which is over budget.

Answer to Question 19

a. Each cup makes $1.20 - 0.35 = 0.85$ pounds, then take off the £15: $P = 0.85n - 15$.
b. $n = 20$: $17 - 15 =$**£2**. $n = 40$: **£19**. $n = 60$: **£36**. $n = 80$: **£53**.
c. $0.85n - 15 = 100$, so $0.85n = 115$ and $n = 135.3$. We must round up to reach £100: **136 cups**.
d. $0.85 \times 18 - 15 = 15.30 - 15 = 0.30$. The profit is 30p, so 18 cups do just cover the costs. The teacher is right.

Answer to Question 20

a. $10 \times 10 =$**100** and $9 \times 11 =$**99**. The product of the two neighbours is 1 less than the square.
b. With $n = 10$, $9 \times 11 = (n - 1)(n + 1) = n^2 + n - n - 1 = n^2 - 1$.
c. Expanding gives $n^2 + n - n - 1$; the $+n$ and $-n$ cancel, leaving $n^2 - 1$, which is always one less than n^2.
d. $49 \times 51 = (50 - 1)(50 + 1) = 50^2 - 1 = 2500 - 1 =$**2 499**.

This result is called the difference of two squares. It lets you multiply two numbers that are equally spaced either side of a round number very quickly in your head.