Lesson - Decimals and percentages

Year: Year 7

Fractions, decimals and percentages are three ways of writing the same amount. In this chapter you will convert between them and use percentages to solve problems.

Starter activity

A shop advertises "30% off everything". A pair of jeans costs £45 before the sale. Work out the sale price in two different ways: first by finding 30% and subtracting, and second by finding 70% directly. Which method do you prefer and why?

Decimals, fractions and percentages

Definition: percentage

A **percentage** is a number of parts per hundred. 37% means $\frac{37}{100}$ or 0.37. The symbol % means “out of 100”.

Key fact: common equivalents

$\frac{1}{2} = 0.5 = 50\%$, $\frac{1}{4} = 0.25 = 25\%$, $\frac{3}{4} = 0.75 = 75\%$, $\frac{1}{5} = 0.2 = 20\%$, $\frac{1}{10} = 0.1 = 10\%$, $\frac{1}{3} = 0.\dot{3} = 33.\dot{3}\%$.

Converting between the forms

Method: converting between fractions, decimals and percentages

1. Fraction to decimal: divide the numerator by the denominator.
2. Decimal to percentage: multiply by 100.
3. Percentage to decimal: divide by 100.
4. Percentage to fraction: write over 100 and simplify.

Example 1

Write $\frac{3}{8}$ as a decimal and a percentage.

$3 \div 8 = 0.375$. Multiply by 100: 37.5%.

Example 2

Write 45% as a fraction in its simplest form.

$\frac{45}{100} = \frac{9}{20}$, dividing both by 5.

Finding a percentage of an amount

Rule: percentage of an amount

Method 1: build from 10% and 1%. Method 2: use a decimal multiplier. Divide the percentage by 100 and multiply by the amount: 35% of 80 is 0.35×80.

Example 3

Find 35% of £80.

10% = £8, so 30% = £24 and 5% = £4. Total £28. Check: $0.35 \times 80 = 28$.

Example 4

Find 12.5% of 64.

12.5% is $\frac{1}{8}$, and $64 \div 8 = 8$.

One quantity as a percentage of another

Rule: expressing as a percentage

Write the first quantity as a fraction of the second, then multiply by 100. $\frac{\text{part}}{\text{whole}} \times 100$.

Example 5

A pupil scores 34 out of 40. What percentage is that?

$\frac{34}{40} \times 100 = 85\%$.

Percentage increase and decrease

Rule: multipliers

To increase by $p\%$, multiply by $1 + \frac{p}{100}$. To decrease by $p\%$, multiply by $1 - \frac{p}{100}$. For

example, a 20% increase is $\times 1.2$ and a 15% decrease is $\times 0.85$.

Example 6

A £60 jacket has 15% off. Find the sale price.

15% of 60 = £9. Sale price $60 - 9 =$£51. Or $60 \times 0.85 = 51$.

Example 7

A population of 8 000 grows by 5%. Find the new population.

$8000 \times 1.05 = 8400$.

Watch out

Do not just add the percentage to the amount. Increasing 50 by 20% gives 60, not 70. A 10% rise followed by a 10% fall does not return to the start.

Calculating with decimals

To add and subtract decimals, line up the decimal points. To multiply, ignore the points, multiply, then count the decimal places in the question. To divide by a decimal, multiply both numbers by 10 or 100 until the divisor is a whole number.

Example 8

Work out 3.6×0.4 and $7.2 \div 0.9$.

$36 \times 4 = 144$, and there are 2 decimal places, so 1.44. For the division, multiply both by 10: $72 \div 9 = 8$.

Comparing amounts using percentages

Example 9

Shop A sells a pen for £2.40 with 25% off. Shop B sells the same pen for £2.00 with 10% off. Which is cheaper?

A: $2.40 \times 0.75 = 1.80$. B: $2.00 \times 0.90 = 1.80$. They cost the same.

Useful facts about percentages

Note

Percentages can be more than 100%. 150% of a number is one and a half times it. Percentages are more useful than fractions for comparing, because they use the same

base of 100.

Recurring decimals and percentage problems

Some fractions give recurring decimals. $\frac{1}{3} = 0.333\ldots$ and $\frac{2}{3} = 0.666\ldots$. A dot above a digit shows that it repeats: $0.\dot{3}$. Percentages can have fraction or decimal parts, such as $33\frac{1}{3}\%$ or 12.5%.

Example 10

Put $\frac{2}{5}$, 0.42, 38% and $\frac{3}{8}$ in order, smallest first.

Change all to decimals: 0.4, 0.42, 0.38 and 0.375. The order is $\frac{3}{8}$, 38%, $\frac{2}{5}$, 0.42.

Example 11

A TV costs £350 with 20% off. A further 10% is then taken off the sale price. Find the final price.

$350 \times 0.8 = 280$. Then $280 \times 0.9 = 252$. The final price is £252. This is not the same as 30% off (£245).

Example 12

A shop buys a phone for £200 and sells it for £260. What is the percentage profit?

Profit $= 60$. $\frac{60}{200} \times 100 = 30\%$.

Note

Percentage change $= \frac{\text{change}}{\text{original}} \times 100$. Always divide by the original amount.

Common mistakes

- Writing 0.5 as 5%. It is 50%.
- Finding 10% by dividing by 100.
- Forgetting to simplify fractions.
- Dividing by the wrong number in “one quantity as a percentage of another”: always divide by the whole.

Worked example in context

A phone costs £240. The price rises by 15%. What is the new price?

Multiplier: $1 + 0.15 = 1.15$. New price: $240 \times 1.15 = 276$. The new price is £276.

Common mistakes

- Adding percentages from different starting amounts.
- Confusing a percentage with a percentage point.
- Putting the decimal point in the wrong place when multiplying decimals.

Did you know?

The symbol % comes from the Italian "per cento", meaning "per hundred". Percentages are used on food labels, weather forecasts ("40% chance of rain"), football statistics and phone batteries. Shops started to use "% off" signs widely in the 20th century.

Choosing a method and checking it

There is usually more than one way to answer a percentage or decimal question, and good mathematicians choose the method that suits the numbers. Without a calculator, building percentages from 10%, 5% and 1% is quick and reliable. With a calculator, the multiplier method is faster and avoids mistakes when there are several steps. Whatever you choose, finish by asking yourself whether the answer is sensible.

Method: estimating before you calculate

1. Round every number to one significant figure, for example $4.83 \times 19.6 \approx 5 \times 20 = 100$.
2. Work out the rough answer in your head.
3. Do the exact calculation, on paper or with a calculator.
4. Compare the two answers. If the exact answer is ten times too big or too small, a decimal point has moved.

Example 1

Work out 0.3×0.04 without a calculator.

Ignore the decimal points and multiply the whole numbers: $3 \times 4 = 12$. Now count the decimal places in the question: 0.3 has one and 0.04 has two, so the answer needs three decimal places. Starting from 12 we move the point three places to the left, which gives 0.012. A quick check: we are taking a small part (0.3) of an already small number (0.04), so the answer should be smaller than 0.04, and it is.

Example 2

Work out $1.8 \div 0.03$.

Dividing by a decimal is awkward, so we multiply both numbers by 100 to make the divisor a whole number. The division $1.8 \div 0.03$ becomes $180 \div 3$, which equals 60. Multiplying both numbers by the same amount does not change the answer, in the same way that $\frac{1.8}{0.03} = \frac{180}{3}$ are equivalent fractions. Notice that dividing by a number smaller than 1 makes the answer bigger.

Example 3

A phone costs £240 after a 20% discount. What was the original price?

This is a reverse percentage. After a 20% discount the customer pays 80% of the original price, so 80% corresponds to £240. Then 10% is $240 \div 8 = 30$, and 100% is $30 \times 10 = 300$. The original price was £300. A common mistake is to add 20% of £240 to get £288, but the 20% was taken from the original price, not from £240. Check: 20% of £300 is £60, and $300 - 60 = 240$.

Watch out

Multiplying does not always make a number bigger, and dividing does not always make it smaller. When you multiply by a number between 0 and 1, such as 0.5 or 0.25, the answer is smaller than the number you started with, because you are taking only a part of it. When you divide by a number between 0 and 1, the answer is bigger: $6 \div 0.5 = 12$, because there are twelve halves in six. Keep this in mind when you check whether an answer is sensible, and never assume that the answer to a multiplication must be the largest number on the page.

Note

Percentages are useful for comparing amounts that are out of different totals. A score of 18 out of 24 and a score of 21 out of 30 are hard to compare directly, but as percentages they are 75% and 70%, so the first is better. In the same way, shops, banks and newspapers use percentages because they make comparisons fair.

Did you know?

The percent sign % grew out of the Italian phrase *per cento*, meaning “for each hundred”. Merchants in the 15th century wrote it as “p cento”, and over the years the words were squeezed into the symbol we use today. Decimals, on the other hand, became popular after the Flemish mathematician Simon Stevin explained them in a short book in 1585.

Otto's tip

Percentages: find 10% first, then build any other percentage from it. It is much quicker than you expect!