Lesson - Angles and parallel lines

Year: Year 7

Angles are measured in degrees. In this chapter you will use angle facts to find missing angles, in triangles, on lines and with parallel lines, and explain each step with a reason.

Starter activity

Look at a clock face. At 3 o'clock the hands make a right angle. At what other time on the hour is the angle between the hands exactly 90°? What angle do the hands make at 2 o'clock? Explain your method using a full turn of 360° and the 12 divisions of the clock.

Measuring and naming angles

Definition: angle types

An **acute** angle is less than 90°. A **right angle** is exactly 90°. An **obtuse** angle is between 90° and 180°. A **reflex** angle is between 180° and 360°.

We name an angle with three letters. $\angle ABC$ is the angle at B between the lines BA and BC. Measure with a protractor, with its centre on the vertex and one arm on the zero line.

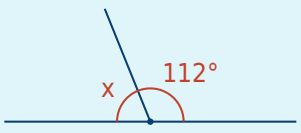
Angle facts

Rule: basic angle facts

Angles on a straight line add up to 180°. Angles around a point add up to 360°. Vertically opposite angles are equal.

Example 1

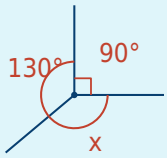
Angles on a straight line are 112° and x. Find x.



$x = 180 - 112 = 68^\circ$ (angles on a straight line add up to 180°).

Example 2

Angles at a point are 90°, 130° and x. Find x.



$x = 360 - 90 - 130 = 140^\circ$ (angles at a point add up to 360°).

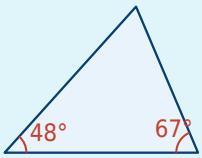
Triangles

Rule: angles in a triangle

The angles in a triangle add up to 180°. An equilateral triangle has three angles of 60°. An isosceles triangle has two equal sides and the angles opposite them are equal (the base angles). A right-angled triangle has one angle of 90°.

Example 3

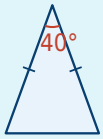
Two angles of a triangle are 48° and 67°. Find the third.



$180 - 48 - 67 = 65^\circ$.

Example 4

An isosceles triangle has an apex angle of 40°. Find the base angles.



Not drawn accurately

The base angles are equal, so each is $(180 - 40) \div 2 = 70^\circ$.

Quadrilaterals

Rule: angles in a quadrilateral

The angles in a quadrilateral add up to 360°, because any quadrilateral can be split into two triangles.

Example 5

Three angles of a quadrilateral are 95°, 110° and 70°. Find the fourth.

$95 + 110 + 70 = 275$, so the fourth angle is $360 - 275 = 85^\circ$.

Parallel lines

Definition: parallel lines and a transversal

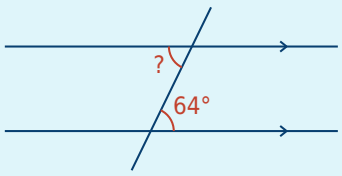
Parallel lines stay the same distance apart and never meet. We mark them with arrows. A line that crosses them is a **transversal**.

Key fact: parallel line angles

Corresponding angles are equal (they make an F shape). **Alternate angles** are equal (they make a Z shape). **Co-interior angles** add up to 180° (they make a C or U shape).

Example 6

Two parallel lines are crossed by a transversal and one angle is 64°. Find the corresponding, alternate and co-interior angles.



The corresponding angle is 64°. The alternate angle is 64°. The co-interior angle is $180 - 64 = 116^\circ$.

Method: finding angles with parallel lines

1. Look for the F, Z or C shape that links the angle you know with the angle you want.
2. Use the matching rule (equal, equal or add to 180°).
3. Write the reason beside the angle.

Using algebra with angles

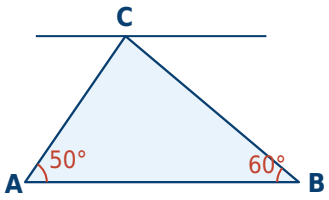
Example 7

Angles on a straight line are $2x$ and $x + 30$. Find x and both angles.

$2x + x + 30 = 180$, so $3x = 150$ and $x = 50$. The angles are $2 \times 50 = 100^\circ$ and $50 + 30 = 80^\circ$. Check: $100 + 80 = 180$.

A multi-step problem

Triangle ABC has a line drawn through C parallel to AB. The alternate angles are equal, and the angles on a straight line add up to 180°. This proves that the angles in a triangle add up to 180°.



Writing the reason beside each angle earns marks in exams and helps you spot mistakes.

Example 8

In a triangle ABC, angle A is 50° and angle B is 60°. The line CD extends BC beyond C. Find the exterior angle at C.

Angle ACB is $180 - 50 - 60 = 70^\circ$. The exterior angle is on a straight line with it, so $180 - 70 = 110^\circ$. This equals $50 + 60$, the sum of the two opposite interior angles.

Common mistakes

- Mixing up 180° (straight line, triangle) and 360° (point, quadrilateral).
- Measuring a diagram that says “not drawn accurately”.
- Confusing alternate and co-interior angles.
- Giving an answer without a reason.

Watch out

Do not assume lines are parallel unless the diagram marks them with arrows. Co-interior angles add up to 180°; they are not equal.

Worked example in context

A ladder leans against a wall making an angle of 70° with the ground. What angle does it make with the wall?

The wall is vertical so the triangle is right-angled: $180 - 90 - 70 = 20$. The ladder makes an angle of 20° with the wall.

Common mistakes

- Using 360° for angles on a straight line instead of 180°.
- Confusing alternate and co-interior angles.
- Not giving a reason, such as "angles in a triangle add to 180°".

Did you know?

Ancient Babylonians divided a circle into 360 parts about 4 000 years ago, probably because their number system was based on 60 and because a year has about 360 days. That is why we still use 360 degrees today.

Giving reasons in angle problems

In angle questions the number is only half of the answer. Examiners also want to know *why* the angle has that size, so every step of your working should end with a short reason written in words. A good reason names the angle fact you used, for example “angles on a straight line add up to 180°” or “alternate angles are equal”. Reasons such as “because it looks like it” or “I measured it” do not score marks, because diagrams are usually not drawn accurately.

Method: solving a multi-step angle problem

1. Copy or sketch the diagram and mark every angle you are given.
2. Look for the angle facts that link a known angle to an unknown one: straight lines, points, triangles, quadrilaterals, vertically opposite angles and parallel lines.
3. Work out one new angle at a time, write it on the diagram and write the reason next to your calculation.
4. Keep going until you reach the angle you were asked for, then check that the angles in every triangle add up to 180°.

Example 1

In triangle PQR, angle P is 64° and angle Q is 58°. The side QR is extended beyond R to a point S. Find angle PRS.

First we find the third angle of the triangle. The angles in a triangle add up to 180°, so angle PRQ is $180 - 64 - 58 = 58^\circ$. Next we use the straight line QRS: angles on a straight line add up to 180°, so angle PRS is $180 - 58 = 122^\circ$. Notice that 122° is exactly $64 + 58$. This is the exterior angle fact: an exterior angle of a triangle equals the sum of the two opposite interior angles.

Example 2

Two parallel lines are crossed by a transversal. One angle is $2x + 15$ and its

co-interior angle is $3x - 5$. Find x.

Co-interior angles lie between the parallel lines on the same side of the transversal, and they add up to 180°. So we form the equation $2x + 15 + 3x - 5 = 180$. Collecting like terms gives $5x + 10 = 180$, then $5x = 170$ and $x = 34$. The two angles are $2 \times 34 + 15 = 83^\circ$ and $3 \times 34 - 5 = 97^\circ$. They add up to 180°, which confirms the answer.

Example 3

Three angles of a quadrilateral are equal and the fourth angle is 87°. Find each of the equal angles.

The angles in a quadrilateral add up to 360°. If we call each equal angle a, then $3a + 87 = 360$. Subtracting 87 gives $3a = 273$, so $a = 91^\circ$. Check: $91 + 91 + 91 + 87 = 360$.

Note

Recognising angle pairs is easier if you look for letters. Corresponding angles sit in an F shape, alternate angles sit in a Z shape and co-interior angles sit in a C (or U) shape. Use the letters to spot the pairs, but in your written reason always use the proper names: corresponding, alternate and co-interior.

Watch out

Parallel line facts only work when the lines really are parallel. Look for the arrow marks on the lines or the word “parallel” in the question. If the lines are not parallel, corresponding angles are not equal and co-interior angles do not add up to 180°. In that case you can only use the facts about straight lines, points and triangles.

Did you know?

The idea of splitting a full turn into 360 degrees comes from the ancient Babylonians, who counted in sixties. The number 360 is very convenient because it can be divided exactly by 2, 3, 4, 5, 6, 8, 9, 10, 12 and many other numbers, so halves, thirds, quarters and sixths of a turn are all whole numbers of degrees.

Otto's tip

Look for the letters! Z means alternate (equal), F means corresponding (equal), and C means co-interior (add to 180°).