Lesson - Algebraic notation and simplifying expressions

Year: Year 7

Algebra is the language of mathematics. Letters stand for numbers, which lets us write rules that work every time. In this chapter you will use algebraic notation, substitute values and simplify expressions.

Starter activity

Maya is buying a snack and a drink at the school canteen. A snack costs £s and a drink costs £d. Write an expression for the cost of 2 snacks and 3 drinks. Then work out the cost if a snack is 80p and a drink is 60p.

Why we use algebra

Suppose a taxi charges £3 plus £2 per mile. Instead of writing a new sum for every journey, we write one rule using a letter: the cost is $2m + 3$ pounds for m miles. The letter m is a **variable** because its value can change.

Algebraic notation

Definition: algebraic notation

Algebraic notation is the shorthand used in algebra. We leave out multiplication signs, so $3 \times a$ is written $3a$ and $a \times b$ is written ab. We write division as a fraction, so $a \div b$ is $\frac{a}{b}$. Also $a \times a = a^2$.

Rule: notation rules

Write the number before the letter ($5x$, not $x5$). Write $1 \times a$ simply as a. Write letters in alphabetical order in a product (abc). A product like $2 \times a \times a$ is written $2a^2$.

Example 1

Write $y \times 4 \times x$ using algebraic notation.

Put the number first and the letters in alphabetical order: $4xy$.

Terms, expressions and coefficients

Definition: term, expression and coefficient

A **term** is a number, a letter, or numbers and letters multiplied together, such as $7x$.

An **expression** is made of terms joined by + or −, such as $3x + 5$. The **coefficient** is the number multiplying the letter, so the coefficient of $7x$ is 7. A term with no letter, such as 5, is a **constant**.

Example 2

In $4x^2 - 3x + 7$, name the terms and the coefficient of x.

The terms are $4x^2$, $-3x$ and 7. The coefficient of x is -3, because the sign belongs to the term.

Substitution

Definition: substitution

Substitution means replacing letters with numbers and then working out the value.

Method: substituting values

1. Write the expression.
2. Replace each letter with its number, in brackets if it is negative.
3. Work out the value using BIDMAS: brackets, indices, division and multiplication, then addition and subtraction.

Example 3

Find the value of $3a + 2b$ when $a = 4$ and $b = 5$.

$3 \times 4 + 2 \times 5 = 12 + 10 = 22$.

Example 4

Find the value of $x^2 - 3x$ when $x = -2$.

$(-2)^2 - 3 \times (-2) = 4 + 6 = 10$.

Watch out

Squaring only affects the letter next to it. $2x^2$ means $2 \times x \times x$, not $(2x)^2$. If $x = 3$, $2x^2 = 18$ but $(2x)^2 = 36$. A negative number must go in brackets before it is squared.

Simplifying by collecting like terms

Definition: like terms

Like terms have exactly the same letters raised to the same powers. $3x$ and $5x$ are like

terms. $3x$ and $3x^2$ are not, and neither are $3x$ and $3y$.

Rule: collecting like terms

To simplify an expression, add or subtract the coefficients of like terms. Keep the sign in front of each term with it. $3x + 5x = 8x$ and $7y - 2y = 5y$.

Method: simplifying an expression

1. Circle or underline each group of like terms, including its sign.
2. Add or subtract the coefficients in each group.
3. Write the answer, with the x terms, then y terms, then numbers.

Example 5

Simplify $5a + 3b - 2a + 4b$.

The a terms: $5a - 2a = 3a$. The b terms: $3b + 4b = 7b$. The answer is $3a + 7b$.

Example 6

Simplify $4x^2 + 3x - x^2 + 5x + 6$.

x^2 terms: $4x^2 - x^2 = 3x^2$. x terms: $3x + 5x = 8x$. The constant is 6. The answer is $3x^2 + 8x + 6$.

Multiplying and dividing terms

Rule: multiplying and dividing terms

To multiply terms, multiply the numbers and then the letters: $3a \times 4b = 12ab$ and $2x \times 3x = 6x^2$. To divide, divide the numbers and cancel the letters: $\frac{12ab}{4a} = 3b$.

Example 7

Simplify $5p \times 2q$ and $\frac{18xy}{6x}$.

$5p \times 2q = 10pq$. For the fraction, $18 \div 6 = 3$ and the x cancels, leaving $3y$.

Forming expressions

Example 8

A pencil costs p pence and a rubber costs 15 pence more. Write an

expression for the cost of 3 pencils and 2 rubbers.

A rubber costs $p + 15$. Three pencils cost $3p$ and two rubbers cost $2(p + 15)$. The total is $3p + 2(p + 15) = 3p + 2p + 30 = 5p + 30$ pence.

Example 9

Write an expression for the perimeter of a rectangle with length l and width w.

Add the four sides: $l + w + l + w = 2l + 2w$.

Note

Algebra follows the same rules as arithmetic. If you are unsure, replace the letters by small numbers and check that both sides give the same value.

Common mistakes

- Adding unlike terms: $3x + 2y$ cannot be simplified to $5xy$.
- Writing $a + a$ as a^2. It is $2a$.
- Losing the sign: in $5x - 3y$ the term is $-3y$.
- Forgetting that x means $1x$.

Watch out

$2a + 3b$ is not $5ab$, and $a + a$ is not a^2. Only like terms can be combined.

Worked example in context

A taxi costs £4 plus £3 per mile. Write an expression for a journey of m miles and use it to find the cost of 5 miles.

Cost = $4 + 3m$. For 5 miles: $4 + 3 \times 5 = 19$. The cost is £19.

Common mistakes

- Writing $3x + 4y = 7xy$: unlike terms cannot be added.
- Thinking that $2x \times 3x = 6x$ when it is $6x^2$.
- Forgetting that $-(x - 3)$ changes both signs.

Did you know?

Algebra gets its name from the Arabic word *al-jabr*, from a book by the mathematician al-Khwarizmi written about 1 200 years ago. The letters x and y became popular after the French mathematician Descartes used them in the 1600s.

Putting it all together

Algebra is a language, and like any language it becomes easier with practice. In this final section we bring together everything you have learnt: writing expressions from words, simplifying them and substituting numbers. Read each worked example slowly, cover the solution and try it yourself first. Then compare your working line by line with the model answer.

Method: turning words into an expression

1. Read the sentence and decide what the letter stands for. Write it down, for example “let n be the number of pupils”.
2. Look for key words: “more than” and “increased by” mean add, “less than” and “fewer” mean subtract, “times”, “each” and “per” usually mean multiply, “shared” and “split” mean divide.
3. Write each part of the sentence as a term, then join the terms with the correct signs.
4. Simplify by collecting like terms and check your expression by substituting an easy number such as 1 or 10.

Example 10

Lily has x stickers. Tom has 4 fewer stickers than Lily and Ana has three times as many as Lily. Write a simplified expression for the total number of stickers.

Lily has x stickers. “4 fewer than Lily” means we subtract 4, so Tom has $x - 4$. “Three times as many” means we multiply by 3, so Ana has $3x$. The total is $x + (x - 4) + 3x$. The like terms in x are x, x and $3x$, which make $5x$, and the number term is -4. The total number of stickers is $5x - 4$. Check with $x = 10$: Lily 10, Tom 6, Ana 30, total 46, and $5 \times 10 - 4 = 46$. The expression works.

Example 11

Simplify $6a + 4b - 2a + b - 3$ and then find its value when $a = 2$ and $b = 5$.

First we group the like terms together, taking each term with the sign in front of it: $6a - 2a$ gives $4a$, and $4b + b$ gives $5b$. The number -3 has no like term, so it stays as it is. The simplified expression is $4a + 5b - 3$. Now we substitute: $4 \times 2 + 5 \times 5 - 3 = 8 + 25 - 3 = 30$. It is a good idea to check by substituting into the original expression too: $12 + 20 - 4 + 5 - 3 = 30$. Both give 30, so our simplifying is correct.

Example 12

Explain why $2x^2$ and $(2x)^2$ are different when $x = 3$.

In $2x^2$ only the x is squared, so we work out $3^2 = 9$ first and then double it to get 18.

In $(2x)^2$ the bracket tells us to work out $2x = 6$ first and then square the whole bracket, giving 36. The order of operations (BIDMAS) decides what happens first, so the two expressions are different even though they use the same symbols.

Watch out

When you substitute a negative number, always put it in brackets. For $x = -3$, x^2 is $(-3)^2 = 9$, not -9. Writing the brackets on your page stops the minus sign from getting lost and is exactly what examiners like to see in your working.

Checking your answers

A quick check can save you marks. After simplifying an expression, choose a simple value for each letter, for example 2 or 3, and substitute it into both the original expression and your answer. If the two results are the same, your simplification is almost certainly right. If they are different, look again at the signs, because most mistakes come from a minus sign that has been forgotten. You can also check that you have not mixed unlike terms: count the letters in each term and make sure only identical letter parts have been combined.

Did you know?

Spreadsheets use algebra all the time. When you type a formula such as =3*A1+2 into a cell, the cell reference A1 behaves exactly like a letter in algebra: the spreadsheet substitutes whatever number is in A1 and works out the value for you. Every time the number in A1 changes, the answer updates automatically, just as an expression gives a new value for every new value of x.

Otto's tip

Think of letters as objects. $3x + 2x$ is 3 apples plus 2 apples, which is 5 apples. But apples plus bananas stay apples plus bananas!