Answers - Algebraic notation and simplifying expressions

Year: Year 7

Answer to Question 1

a. $a \times b$ is written without the multiplication sign, letters in alphabetical order: ab.
b. The number is written before the letter and the sign is left out: 5 times n is $5n$.
c. A letter multiplied by itself is a square, so $p \times p$ is p^2.
d. Number first, then the letters in order, no signs: $3xy$.
e. In algebra we write division as a fraction: $m \div 4$ is $\frac{m}{4}$.
f. The whole bracket is multiplied by 5, and the number goes in front: $5(x + 2)$.

Answer to Question 2

a. Each pen costs £p and there are 7 pens, so we multiply: the cost is **£7p**.
b. In 6 years she will be 6 years older, so we add 6 to her age: $n + 6$.
c. 4 bags of s sweets give $4 \times s = 4s$ sweets, then we add the 3 loose ones: $4s + 3$.
d. The width is 5 cm less than l, so it is $l - 5$ **cm**. The perimeter is two lengths plus two widths: $2l + 2(l - 5) = 2l + 2l - 10 = 4l - 10$ **cm**.

Answer to Question 3

a. $3a$ and $5a$ are like terms: $3 + 5 = 8$, so the answer is $8a$.
b. All the terms are in x: $7 - 2 + 4 = 9$, giving $9x$.
c. Collect the a terms: $4a + 2a = 6a$. Collect the b terms: $3b - b = 2b$. Answer: $6a + 2b$.
d. p terms: $5p - 3p = 2p$. Numbers: $2 + 7 = 9$. Answer: $2p + 9$.
e. x^2 terms: $6x^2 - 2x^2 = 4x^2$. x terms: $3x + x = 4x$. They are unlike terms, so the answer is $4x^2 + 4x$.
f. ba is the same as ab, so all three are like terms: $3 + 4 - 2 = 5$. Answer: $5ab$.

Answer to Question 4

a. Multiply the numbers $3 \times 4 = 12$, then the letters $a \times b = ab$: $12ab$.
b. $2 \times 5 = 10$ and $x \times x = x^2$: $10x^2$.
c. A positive times a negative is negative: $6 \times (-2) = -12$, letters pq: $-12pq$.
d. Divide the numbers $12 \div 3 = 4$ and cancel the common x: $4y$.
e. $20 \div 5 = 4$ and $a^2 \div a = a$: $4a$.

f. A negative times a negative is positive: $4 \times 3 = 12$ and $m \times m = m^2$, so $12m^2$.

Answer to Question 5

a. The terms are separated by + and − signs: $5x^2$, $-3x$ and 7. There are **3** terms.
b. The number in front of x is the coefficient, together with its sign: -3.
c. The constant term is the number with no letter: **7**.
d. Like terms have exactly the same letter part. Only $4x$ and $-x$ are both in x, so they are the like terms.
e. Any three terms that collect to $3x + 2y$ work, for example $4x - x + 2y$: $4x - x = 3x$, giving $3x + 2y$.

Answer to Question 6

a. y is multiplied by itself three times, so we use the power 3: y^3.
b. There are two a's, so $a \times a = a^2$, and the 2 goes in front: $2a^2b$.
c. The bracket is squared: $3x \times 3x = 9x^2$, so the answer is $9x^2$.
d. Only x is squared: $4^2 = 16$, then $3 \times 16 =$**48**.
e. Work out the bracket first: $3 \times 4 = 12$, then $12^2 =$**144**. This is different from part (d), because the bracket changes the order.

Answer to Question 7

a. 3 child tickets cost $3c$ and 2 adult tickets cost $2a$. Total: $3c + 2a$ pounds.
b. The second family pays $2c + 3a$. Both families together: $(3c + 2a) + (2c + 3a)$.
c. Collect like terms: $3c + 2c = 5c$ and $2a + 3a = 5a$, giving $5c + 5a$.
d. Substitute $c = 6$, $a = 9$: $5 \times 6 + 5 \times 9 = 30 + 45 =$**£75**. Both families pay £75 in total.

Answer to Question 8

a. Change pence to pounds: 5p is £0.05 and 8p is £0.08. The bill is the fixed £12 plus the extras: $12 + 0.05t + 0.08m$.
b. Substitute: $12 + 0.05 \times 40 + 0.08 \times 25 = 12 + 2 + 2 =$**£16**.
c. His sister only pays for texts: $12 + 0.05 \times 60 = 12 + 3 =$**£15**.
d. A formula starts with the subject and an equals sign: $B = 12 + 0.05t + 0.08m$.

Answer to Question 9

a. Perimeter = two lengths + two widths: $2x + 2(x - 3) = 2x + 2x - 6 = 4x - 6$

metres.
b. Area = length × width: $x(x-3) = x^2 - 3x$ square metres.
c. Substitute $x = 10$ into $4x - 6$: $40 - 6 =$**34 m**.
d. Cost = 8 × perimeter: $8(4x-6) = 32x - 48$. For $x = 10$: $320 - 48 =$**£272**, which is also 8×34.

Answer to Question 10

a. £150 fixed plus £4 for each of the n pupils: $150 + 4n$.
b. Each pupil pays £7 entry, so n pupils cost $7n$.
c. Add the two costs: $150 + 4n + 7n$. Collect $4n + 7n = 11n$: $150 + 11n$.
d. Substitute $n = 30$: $150 + 11 \times 30 = 150 + 330 =$**£480**.
e. We need $150 + 11n \leq 500$, so $11n \leq 350$ and $n \leq 31.8$. Pupils must be whole, and 32 would cost £502, so the largest number is **31 pupils**.

Answer to Question 11

a. The brother gets £2 more than Priya, so we add 2: $x + 2$.
b. The cousin gets twice Priya's amount: $2x$.
c. Total $= x + (x+2) + 2x$. Collect the x terms $1 + 1 + 2 = 4$: $4x + 2$.
d. Substitute $x = 5$: $4 \times 5 + 2 = 20 + 2 =$**£22**.
e. Yes. From part (d), when $x = 5$ the total is exactly £22, so Priya gets £5, her brother £7 and her cousin £10.

Answer to Question 12

a. Start with n. Multiply by 3: $3n$. Add 6: $3n + 6$. Divide by 3: $\frac{3n+6}{3} = n + 2$. Take away n: $n + 2 - n$.
b. In $n + 2 - n$ the n terms cancel, because $n - n = 0$. What is left is **2**, whatever number you start with.
c. Any trick where the starting number cancels works, for example: think of n, double it to get $2n$, add 10 to get $2n + 10$, halve to get $n + 5$, subtract n to get **5**.
d. Start with 7: double 14, add 10 gives 24, halve 12, subtract 7 gives **5**. The trick works.

Answer to Question 13

a. Each new square needs 3 more matchsticks: 4, 7, 10, so the next is $10 + 3 =$**13**.
b. The pattern goes up by 3, so it starts with $3n$. For $n = 1$, $3n = 3$ and we need 4, so add 1: $3n + 1$.
c. Substitute $n = 20$: $3 \times 20 + 1 =$**61** matchsticks.

d. We need $3n + 1 \leq 100$, so $3n \leq 99$ and $n \leq 33$. Check: $3 \times 33 + 1 = 100$. She can make **33 squares**.

Answer to Question 14

a. Add all six sides: x terms $3x + 2x + x + x + 2x + x = 10x$, numbers $2 - 2 + 2 = 2$. Perimeter: $10x + 2$ cm.
b. Substitute $x = 5$: $10 \times 5 + 2 = 50 + 2 =$**52 cm**.
c. Form the equation $10x + 2 = 74$. Subtract 2: $10x = 72$. Divide by 10: $x =$**7.2**. Check: $72 + 2 = 74$.

Answer to Question 15

a. Area = base × height. A: $2a \times h = 2ah$, B: $3a \times h = 3ah$, C: $a \times h = ah$.
b. The three areas are like terms: $2ah + 3ah + ah = 6ah$.
c. The big rectangle has base $2a + 3a + a = 6a$ and height h. Perimeter: $2(6a) + 2h = 12a + 2h$.
d. Area: $6 \times 4 \times 5 =$**120 cm²**. Perimeter: $12 \times 4 + 2 \times 5 = 48 + 10 =$**58 cm**.

Answer to Question 16

a. Company A: £3 plus £2 per mile, $3 + 2m$. Company B: £1 plus £2.50 per mile, $1 + 2.5m$.
b. Substitute 2, 4, 6, 8. A: 7, 11, 15, 19. B: 6, 11, 16, 21.
c. Set the costs equal: $3 + 2m = 1 + 2.5m$. Subtract $2m$ and 1: $2 = 0.5m$, so $m = 4$. Both cost £11 at **4 miles**.
d. A: $3 + 20 = 23$, B: $1 + 25 = 26$. **Company A** is cheaper by $26 - 23 =$**£3**.

Answer to Question 17

a. 3 kg of apples $3a$, 2 kg of bananas $2b$, 1 kg of grapes g: $3a + 2b + g$.
b. 1 kg apples, 4 kg bananas, 2 kg grapes: $a + 4b + 2g$.
c. Collect like terms: $3a + a = 4a$, $2b + 4b = 6b$, $g + 2g = 3g$: $4a + 6b + 3g$.
d. Mrs Clarke: $6 + 3 + 4 =$£13. Mr Dean: $2 + 6 + 8 =$£16. Total £29. Check with (c): $8 + 9 + 12 = 29$, so it agrees.

Answer to Question 18

a. Zoe added unlike terms. $3x$ and $4y$ have different letters, so $3x + 4y$ cannot be simplified.
b. Adam treated $5a$ and $2a^2$ as like terms. They are not, so $5a - 2a^2$ stays as it is.

c. Beth forgot that $x \times x = x^2$. Correct: $2 \times 3 = 6$, so $6x^2$.
d. Carl only changed one sign. The minus changes both terms: $-x + 3$.

Answer to Question 19

a. Twice packet A is $2x$, and 3 more gives $2x + 3$.
b. Half of packet B: $\frac{2x + 3}{2} = x + 1.5$. Packet C has $x + 1.5$ sweets.
c. Total $= x + (2x + 3) + (x + 1.5)$. x terms: $4x$; numbers: 4.5. Answer: $4x + 4.5$.
d. For $x = 5$: $20 + 4.5 =$**24.5**. No: $x + 1.5$ always ends in .5, so packet C is never whole and the situation is impossible for real sweets.

Answer to Question 20

a. Add neighbours: $a + 2a = 3a$ and $2a + 3a = 5a$. Middle row: **3a, 5a**.
b. The top block is the sum of the middle row: $3a + 5a =$**8a**.
c. Middle row: $x + y$ and $y + x$. Top: $(x + y) + (y + x) = 2x + 2y$.
d. Middle: $x + 4$ and $4 + x$. Top: $2x + 8 = 24$. Subtract 8: $2x = 16$, so $x =$**8**. Check: $16 + 8 = 24$.