Lesson - The four operations, order of operations and long division

Year: Year 6

In Year 6 you use formal methods to multiply and divide large numbers, follow the order of operations and work with factors and primes.

Starter activity

A baker has 1 764 rolls to pack into boxes of 12. Is there a quick way to find the number of boxes? Could you check it using only multiplication? In this chapter you will use formal written methods for the four operations, learn the rules that tell us which operation to do first, and explore the building blocks of whole numbers: factors, multiples and primes.

Long multiplication

Definition: long multiplication

Long multiplication multiplies by a number with two digits. We multiply by the ones, then by the tens (starting with a zero placeholder), then add.

Method: long multiplication

1. Estimate the answer.
2. Multiply by the ones digit.
3. Write a 0 as a placeholder and multiply by the tens digit.
4. Add the two rows and compare with your estimate.

Example 1

Work out 3 468 × 27.

3 468 × 7 = 24 276. 3 468 × 20 = 69 360. Add: 24 276 + 69 360 = 93 636. Estimate: 3 500 × 30 = 105 000, so it is sensible.

Long division

Definition: long division

Long division divides by a number with two digits. We work from the left and write multiples of the divisor to help.

Method: long division

1. Write out the multiples of the divisor (×1 to ×9).
2. Divide into the first digits, subtract, bring down the next digit.
3. Repeat until all digits are used.
4. Write the remainder as a fraction or decimal if needed.

Example 2

Work out 1 764 ÷ 12.

12 into 17 goes 1, remainder 5. Bring down 6 to get 56. 12 into 56 goes 4, remainder 8. Bring down 4 to get 84. 12 into 84 goes 7. The answer is 147.

Example 3

Work out 950 ÷ 25.

25 × 38 = 950, so the answer is 38.

Order of operations: BIDMAS

Rule: BIDMAS

Work in this order: Brackets, Indices (powers), Division and Multiplication (left to right), Addition and Subtraction (left to right).

Example 4

Work out $3 + 4 \times 5$.

Multiplication first: 4 × 5 = 20. Then 3 + 20 = 23.

Example 5

Work out $(12 - 4) \div 2 + 6^2$.

Brackets: 8. Indices: $6^2 = 36$. Division: 8 ÷ 2 = 4. Addition: 4 + 36 = 40.

Watch out

Do not just work from left to right. 3 + 4 × 5 is 23, not 35.

Factors, multiples and primes

Definition: factor, multiple and prime

A **factor** divides exactly into a number. A **multiple** is in its times table. A **prime** has exactly two factors, 1 and itself.

Rule: common factors and multiples

A common factor divides into two numbers. A common multiple is in both times tables. The highest common factor (HCF) and lowest common multiple (LCM) are the greatest and smallest of these.

Example 6

Find the common factors of 24 and 36, and the lowest common multiple of 4 and 6.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24. Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36. Common: 1, 2, 3, 4, 6, 12. Multiples of 4: 4, 8, 12... Multiples of 6: 6, 12... The LCM is 12.

Checking and using the methods

Rule: inverse operations

Multiplication and division are inverses, and so are addition and subtraction. You can check a division by multiplying the answer by the divisor and adding the remainder. For example, $156 \times 15 + 5 = 2\,340 + 5 = 2\,345$, so $2\,345 \div 15 = 156$ remainder 5 is correct.

Example 7

A bus holds 52 passengers. How many buses are needed for 342 pupils?

$342 \div 52 = 6$ remainder 30. Six buses hold only 312 pupils, so 30 pupils would be left behind. We must round *up* to **7 buses**. The remainder tells us what to do: sometimes we round up, sometimes we ignore it and sometimes we write it as a fraction or a decimal. Always re-read the question.

Example 8

Work out $3\,000 \div 16$ as a decimal.

$16 \times 187 = 2\,992$, which leaves a remainder of 8. We write $8 \div 16 = 0.5$ as the decimal part, so the answer is **187.5**.

Example 9

Work out $7 + (9 - 3) \times 2^2 - 10 \div 5$.

Brackets: $9 - 3 = 6$. Indices: $2^2 = 4$. Multiplication and division: $6 \times 4 = 24$ and $10 \div 5$

= 2. Now the expression is 7 + 24 − 2. Addition and subtraction from the left: 7 + 24 = 31, then 31 − 2 = **29**.

Example 10

Find the highest common factor of 28 and 42.

Factors of 28: 1, 2, 4, 7, 14, 28. Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42. The common factors are 1, 2, 7 and 14. The HCF is **14**.

Watch out: common mistakes

In long multiplication, forgetting the zero placeholder gives an answer that is ten times too small. In BIDMAS, division does not always come before multiplication: they have equal priority, so work from left to right. For example 24 ÷ 6 × 2 is 4 × 2 = 8, not 24 ÷ 12 = 2. Finally, 1 is not a prime number because it has only one factor, and 2 is the only even prime number.

Square numbers and cube numbers

A **square number** is a number multiplied by itself: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144. We write $7^2 = 49$. A **cube number** is a number multiplied by itself twice: 1, 8, 27, 64, 125. We write $4^3 = 4 \times 4 \times 4 = 64$. Learn these by heart because they are useful in many problems.

Did you know?

The Greek mathematician Eratosthenes invented a method for finding prime numbers more than 2 000 years ago. He wrote the numbers in a grid and crossed out all the multiples of 2, then 3, then 5, and so on. The numbers that were left were the primes. We still call it the "sieve of Eratosthenes".

Otto's tip

Estimate first with rounded numbers. And remember BIDMAS: brackets always come first!