Lesson - Ratio, proportion and percentages

Year: Year 6

Ratio compares amounts. In Year 6 you use ratio language, scale factors and percentages to solve problems.

Starter activity

Zoe and her brother are given 20 sweets to share. Zoe is 12 and her brother is 8, so they decide to share the sweets in the ratio of their ages. How many sweets does each of them get? In this chapter you will learn how to compare quantities with ratio, how to share fairly in a ratio and how to scale shapes and recipes up or down. You will also use percentages to compare things that are different sizes.

What is a ratio?

Definition: ratio

A **ratio** compares two quantities. The ratio 2 : 3 means for every 2 of one thing there are 3 of the other. The order matters.

Rule: simplifying ratios

To simplify a ratio, divide both parts by a common factor. 12 : 18 = 2 : 3 when we divide by 6.

Example 1

In a class there are 12 boys and 18 girls. Write the ratio of boys to girls in its simplest form.

12 : 18 divides by 6 to give 2 : 3.

Proportion and missing values

Definition: proportion

Proportion compares a part with the whole. If 2 out of every 5 sweets are red, the proportion that are red is $\frac{2}{5}$.

Method: finding a missing value

1. Find the scale factor that takes you from one amount to the other.
2. Multiply or divide the other amount by the same scale factor.

Example 2

3 pens cost £2.40. How much do 12 pens cost?

3 × 4 = 12, so multiply the cost by 4: 2.40 × 4 = £9.60.

Sharing in a ratio (unequal sharing)

Method: sharing in a ratio

1. Add the parts of the ratio to find the total number of parts.
2. Divide the amount by the total number of parts to find one part.
3. Multiply to find each share.
4. Check that the shares add up to the whole.

Example 3

Share £60 in the ratio 2 : 3.

2 + 3 = 5 parts. £60 ÷ 5 = £12 a part. Shares: 2 × 12 = £24 and 3 × 12 = £36. Check: 24 + 36 = 60.

Scale factors and similar shapes

Definition: scale factor

A **scale factor** is the number we multiply lengths by to enlarge a shape. Similar shapes have the same angles and their lengths are in the same ratio.

Example 4

A rectangle 4 cm by 3 cm is enlarged by scale factor 5. Find the new size.

4 × 5 = 20 cm and 3 × 5 = 15 cm. The new rectangle is 20 cm by 15 cm.

Watch out

Multiply every length by the scale factor, not just one. Angles do not change.

Percentages for problems and comparison

Key fact: percentage equivalents

50% = $\frac{1}{2}$, 25% = $\frac{1}{4}$, 10% = $\frac{1}{10}$, 1% = $\frac{1}{100}$. Percentages let us compare fractions and amounts fairly.

Example 5

Which is better: 18 out of 20, or 22 out of 25?

18 out of 20 = 90%. 22 out of 25 = 88%. So 18 out of 20 is better.

Using ratio in real life

Key fact: ratio and fractions

A ratio and a fraction are closely connected. If red and blue counters are in the ratio 3 : 5, there are 3 + 5 = 8 parts altogether, so $\frac{3}{8}$ of the counters are red and $\frac{5}{8}$ are blue. A ratio compares parts with parts, while a fraction compares a part with the whole.

Example 6

Write 60 cm : 2 m in its simplest form.

The two quantities must be in the same units. 2 m = 200 cm, so the ratio is 60 : 200. The highest common factor is 20: 60 ÷ 20 = 3 and 200 ÷ 20 = 10. The simplest form is **3 : 10**. Never leave units in a simplified ratio.

Example 7

Sand and cement are mixed in the ratio 3 : 1. How much cement goes with 18 kg of sand?

Sand is 3 parts and the sand mass is 18 kg, so 1 part is 18 ÷ 3 = 6 kg. Cement is 1 part, which is **6 kg**. The total mass of the mix is 18 + 6 = 24 kg.

Example 8

Zoe and her brother share 20 sweets in the ratio 12 : 8. How many does each get?

First simplify the ratio to 3 : 2. There are 3 + 2 = 5 parts. 20 ÷ 5 = 4 sweets a part. Zoe gets 3 × 4 = **12 sweets** and her brother gets 2 × 4 = **8 sweets**. Check: 12 + 8 = 20.

Example 9

A pizza recipe for 4 people uses 300 g of flour. How much flour is needed for 10 people?

Find the amount for one person: 300 ÷ 4 = 75 g. Then multiply by 10: 75 × 10 = **750 g**. This is called the unitary method. Another way is to use a scale factor of 10 ÷ 4 = 2.5, and then 300 × 2.5 = 750 g.

Watch out: common mistakes

Do not mix up ratio and fractions: in the ratio 2 : 3, the fraction for the first part is $\frac{2}{5}$, not $\frac{2}{3}$. When you enlarge a shape, multiply *every* length by the scale factor. Remember that the area is not multiplied by the scale factor but by the scale factor squared: a scale factor of 2 makes the area 4 times bigger.

Drawing a bar model

For sharing in a ratio, draw one box for each part. To share £60 in the ratio 2 : 3, draw 5 boxes. Each box is worth £60 ÷ 5 = £12. Two boxes belong to the first person (£24) and three boxes belong to the second person (£36).

Did you know?

Artists and architects have used ratios for thousands of years. The ancient Greeks believed that the most beautiful rectangles had sides in a special ratio of about 1.6 : 1, which is called the golden ratio. Many famous buildings, such as the Parthenon in Athens, were designed with this ratio in mind.

Otto's tip

Draw a bar model. Split it into the total number of parts and each part is easy to see!