Lesson - Fractions: add, subtract, multiply and divide

Year: Year 6

In Year 6 you add, subtract, multiply and divide fractions. The key skill is finding a common denominator.

Starter activity

Mia has a pizza cut into 12 equal slices. She eats 3 slices and her brother eats 4 slices. What fraction of the pizza have they eaten together? What fraction is left? In this chapter you will learn how to add, subtract, multiply and divide fractions, including mixed numbers, so that you can answer questions like this in your head or on paper.

Equivalent fractions and simplifying

Definition: simplest form

A fraction is in its **simplest form** when the numerator and denominator have no common factor except 1. To simplify, divide both by their highest common factor.

Example 1

Simplify $\frac{18}{24}$.

The highest common factor of 18 and 24 is 6. 18 ÷ 6 = 3 and 24 ÷ 6 = 4. The answer is $\frac{3}{4}$.

Adding and subtracting fractions

Definition: common denominator

A **common denominator** is a multiple of both denominators. The lowest common denominator is the smallest one.

Method: adding or subtracting fractions

1. Find a common denominator.
2. Write equivalent fractions.
3. Add or subtract the numerators.
4. Simplify the answer.

Example 2

Work out $\frac{2}{3} + \frac{1}{4}$.

The common denominator is 12. $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.

Example 3

Work out $\frac{5}{6} - \frac{3}{8}$.

The common denominator is 24. $\frac{20}{24} - \frac{9}{24} = \frac{11}{24}$.

Mixed numbers

Key fact: mixed numbers

To work with mixed numbers, add or subtract the whole numbers and the fractions separately, or turn them into improper fractions first.

Example 4

Work out $1\frac{2}{3} + 2\frac{3}{4}$.

Wholes: 1 + 2 = 3. Fractions: $\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$. Total: $4\frac{5}{12}$.

Example 5

Work out $4\frac{1}{5} - 1\frac{7}{10}$.

Common denominator is 10: $4\frac{2}{10} - 1\frac{7}{10}$. Write $4\frac{2}{10}$ as $3\frac{12}{10}$. Subtract: $3 - 1 = 2$ and $\frac{12}{10} - \frac{7}{10} = \frac{5}{10}$. The answer is $2\frac{1}{2}$.

Note

A mixed number can be written as an improper fraction, for example $2\frac{3}{4} = \frac{11}{4}$. Convert back at the end, and check your answer is sensible with an estimate.

Estimating helps you check. For $1\frac{2}{3} + 2\frac{3}{4}$, round to 2 + 3 = 5, and the answer $4\frac{5}{12}$ is close to 5.

Multiplying fractions

Rule: multiplying fractions

To multiply fractions, multiply the numerators and multiply the denominators: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. Simplify if you can.

Example 6

Work out $\frac{2}{3} \times \frac{3}{5}$.

$\frac{2\times3}{3\times5} = \frac{6}{15} = \frac{2}{5}$.

Example 7

Find $\frac{3}{4}$ of 20.

20 ÷ 4 = 5 and 5 × 3 = 15.

Dividing a fraction by a whole number

Rule: dividing a fraction

To divide a fraction by a whole number, multiply the denominator by that number: $\frac{a}{b} \div n = \frac{a}{b\times n}$.

Example 8

Work out $\frac{3}{4} \div 3$.

$\frac{3}{4\times3} = \frac{3}{12} = \frac{1}{4}$. Check: 3 ÷ 3 = 1, so $\frac{1}{4}$.

Fractions in real problems

Example 9

Ali spends $\frac{1}{3}$ of his money on a book and $\frac{1}{4}$ on lunch. What fraction is left?

$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$. Left: $1 - \frac{7}{12} = \frac{5}{12}$.

Watch out

When adding fractions, do not add the denominators. $\frac{1}{3} + \frac{1}{4}$ is not $\frac{2}{7}$.

More fraction skills

Key fact: fractions are divisions

The fraction $\frac{3}{4}$ means 3 ÷ 4. This is why we find a fraction of an amount by dividing by the denominator and multiplying by the numerator. To find $\frac{3}{4}$ of 20, work out 20 ÷ 4 = 5, then 5 × 3 = 15. A fraction bigger than 1, such as $\frac{11}{4}$, is called an **improper fraction**, and it can be written as the mixed number $2\frac{3}{4}$.

Method: comparing and ordering fractions

1. Find a common denominator for all the fractions.
2. Write each fraction as an equivalent fraction with that denominator.
3. Compare the numerators: the bigger numerator means the bigger fraction.
4. Write the fractions in their original form in the right order.

Example 10

Put $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{7}{12}$ in order, smallest first.

The common denominator is 12. $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$ and $\frac{7}{12}$ stays the same. The numerators are 7, 8 and 9, so the order is $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{4}$.

Example 11

Work out $2\frac{1}{4} \times \frac{2}{3}$.

Change the mixed number to an improper fraction: $2\frac{1}{4} = \frac{9}{4}$. Now multiply: $\frac{9}{4} \times \frac{2}{3} = \frac{18}{12}$. Simplify by dividing top and bottom by 6: $\frac{3}{2} = 1\frac{1}{2}$. Cancelling first is quicker: the 9 and the 3 share a factor of 3, and the 4 and the 2 share a factor of 2.

Example 12

A recipe uses $\frac{3}{4}$ kg of flour. How much flour is needed for 5 recipes?

Multiply the fraction by the whole number: $5 \times \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$ kg. We write the whole number 5 as $\frac{5}{1}$ so that we can multiply the numerators and the denominators.

Example 13

Work out $\frac{3}{5}$ of £45, and what fraction of the money is left.

£45 ÷ 5 = £9 and £9 × 3 = £27. The fraction left is $1 - \frac{3}{5} = \frac{2}{5}$, which is £18. Check: £27 + £18 = £45.

Watch out: common mistakes

Adding the denominators is the most common mistake: $\frac{1}{2} + \frac{1}{3}$ is $\frac{5}{6}$, not $\frac{2}{5}$. Another mistake is multiplying a fraction by a whole number and also multiplying the denominator. Also remember that $\frac{1}{4} \div 3$ is smaller than $\frac{1}{4}$. When you divide, the answer is smaller, not bigger, so check your answer is sensible.

Using a bar model

Draw a bar for the whole amount and divide it into equal parts, one for each unit of

the denominator. Shade the parts that belong to the fraction. A bar model is a good way to see why $\frac{3}{4} \div 3 = \frac{1}{4}$: three shaded parts shared between three people gives one part each.

Did you know?

The ancient Egyptians wrote nearly all their fractions as unit fractions, which have 1 as the numerator. To write $\frac{3}{4}$ they added $\frac{1}{2} + \frac{1}{4}$. A famous papyrus from about 1550 BC shows tables for turning fractions into sums of unit fractions.

Otto's tip

Find the common denominator first. Always simplify your answer at the end!