# Lesson - Algebra: formulae, sequences and missing numbers

Year: Year 6

Algebra uses letters to stand for numbers. In Year 6 you will use formulae, describe sequences and solve missing number problems.

**Starter activity**

A taxi driver charges a fixed fee of £3 and then £2 for every mile. How much would a 6-mile trip cost? What about a 20-mile trip? Writing the cost out each time takes a long time, so mathematicians use letters. In this chapter you will use letters to write rules, work out the value of expressions, find rules for sequences, and solve missing number problems.

## Letters stand for numbers

**Definition: variable**

A **variable** is a letter that stands for a number that can change, such as $n$ or $x$. In algebra, $3n$ means 3 × n and $n + 4$ means 4 more than n.

**Example 1**

**Find the value of $3n + 2$ when $n = 5$.**

Replace n with 5: 3 × 5 + 2 = 15 + 2 = 17.

## Using simple formulae

**Definition: formula**

A **formula** is a rule written with letters that links quantities. The perimeter of a rectangle is $P = 2l + 2w$, and the area is $A = lw$.

**Method: using a formula**

1. Write the formula.
2. Replace each letter with its number.
3. Work out the answer in the right order, and write the unit.

**Example 2**

**Use $A = lw$ to find the area of a rectangle with $l = 9$ cm and $w = 4$ cm.**

A = 9 × 4 = 36 cm².

### Example 3

**A taxi costs £3 plus £2 for each mile. The formula is $C = 2m + 3$. Find the cost of a 6-mile trip.**

C = 2 × 6 + 3 = 12 + 3 = £15.

## Linear number sequences

### Definition: sequence

A **sequence** is a list of numbers that follows a rule. Each number is a **term**. In a linear sequence the terms go up or down by the same amount each time.

### Rule: term-to-term rule

To find the term-to-term rule, find the difference between neighbouring terms. In 5, 9, 13, 17 the rule is “add 4”.

### Example 4

**The first term is 3 and the rule is “add 6”. Write the first five terms.**

3, 9, 15, 21, 27.

## The nth term

### Rule: nth term

For a linear sequence, the nth term is (difference) × n + (adjustment). Multiply the term number by the difference, then add or subtract to match the first term.

### Example 5

**Find the nth term of 5, 8, 11, 14.**

The difference is 3, so start with $3n$: 3, 6, 9, 12. We need 5, 8, 11, 14, which is 2 more each time. The nth term is $3n + 2$. Check: n = 4 gives 14.

### Example 6

**Use $3n + 2$ to find the 20th term.**

3 × 20 + 2 = 62.

## Missing number problems

**Method: solving a missing number problem**

1. Write the problem with a letter for the missing number.
2. Do the inverse operation to both sides.
3. Check by putting your answer back in.

**Example 7**

**Solve $2x + 7 = 19$.**

Subtract 7: 2x = 12. Divide by 2: x = 6. Check: 2 × 6 + 7 = 19.

## Two unknowns

Sometimes there are two unknown numbers. For $a + b = 10$, pairs such as (1, 9), (2, 8) and (5, 5) all work. Find every pair that fits.

**Example 8**

**Find all the whole-number pairs where $a + b = 6$ and a is bigger than b.**

The pairs are (6, 0), (5, 1) and (4, 2).

**Watch out**

$3n$ means 3 × n, not 30 or 3 + n. Do not forget to multiply by the position number, and check your nth term on at least two terms.

## More algebra skills

**Method: finding the nth term of a linear sequence**

1. Find the common difference between the terms.
2. Write the multiples of this difference, starting with the first multiple. For a difference of 4 write $4n$: 4, 8, 12, 16.
3. Compare these multiples with the sequence to see what must be added or subtracted.
4. Write the nth term and test it on the first two terms.

**Example 9**

**Find the nth term of 7, 12, 17, 22, ... and use it to decide whether 100 is in**

**the sequence.**

The difference is 5, so start with $5n$: 5, 10, 15, 20. Add 2 to match 7, 12, 17, 22. The nth term is $5n + 2$. For 100: $5n + 2 = 100$, so $5n = 98$ and $n = 19.6$. This is not a whole number, so **100 is not in the sequence**.

### Example 10

**A plumber charges £30 call-out plus £25 per hour. Write a formula and find the cost of a 3-hour job.**

Let $h$ be the number of hours. The formula is $C = 25h + 30$. For $h = 3$: $C = 25 \times 3 + 30 = 75 + 30 =$ **£105**.

### Example 11

**Solve $5x - 8 = 27$.**

Add 8 to both sides: $5x = 35$. Divide both sides by 5: $x = 7$. Check by substituting: $5 \times 7 - 8 = 35 - 8 = 27$. This is correct, so $x = 7$.

### Example 12

**Find all the whole-number pairs $(x, y)$ so that $x + 2y = 9$.**

Try $y = 0, 1, 2, 3, 4$ and work out $x = 9 - 2y$: (9, 0), (7, 1), (5, 2), (3, 3), (1, 4). When $y = 5$, $x = -1$, which is not a whole number that we can use here, so there are **five pairs**.

### Watch out: common mistakes

The expression $4n$ means 4 × n and is not the number 4 followed by n. When you solve an equation, do the same to both sides: if you subtract 3 on the left, subtract 3 on the right as well. Another mistake is to forget to check: a quick substitution always tells you whether the answer is right.

### Linear and non-linear sequences

In a linear sequence, the difference between neighbouring terms is always the same. In the sequence 1, 4, 9, 16, the differences are 3, 5 and 7, so it is *not* linear. These are the square numbers. Only linear sequences have an nth term of the form (difference) × n + (adjustment).

### Did you know?

The word "algebra" comes from the Arabic word "al-jabr", which appears in the title of a famous book written by the mathematician al-Khwarizmi about 1 200 years ago in Baghdad. His book showed how to solve equations step by step, and his name gave us

the word "algorithm".

**Otto's tip**

Test your rule. If your nth term does not give the first term when n = 1, change the adjustment!