Answers - Algebra: formulae, sequences and missing numbers

Year: Year 6

Answer to Question 1

a. Replace n with 5. Multiplication comes before addition: $3 \times 5 = 15$, then $15 + 2 =$ **17**.
b. Replace n with 6: $4 \times 6 = 24$, then $24 - 3 =$ **21**.
c. Work out $2n$ first: $2 \times 7 = 14$. Then subtract from 20: $20 - 14 =$ **6**. (Doing 20 − 2 first would give the wrong answer.)
d. Brackets first: $3 + 1 = 4$. Then multiply by 5: $5 \times 4 =$ **20**.
e. $\frac{n}{2}$ means $n \div 2$: $10 \div 2 = 5$. Then $5 + 9 =$ **14**.

Answer to Question 2

a. Write $P = 4s$ and replace s with 7: $P = 4 \times 7 =$ **28 cm**. A perimeter is a length, so the unit is cm.
b. lw means $l \times w$: $A = 9 \times 6 =$ **54 cm²**. Area is measured in square units.
c. Multiply before adding: $2 \times 8 = 16$ and $2 \times 5 = 10$, so $P = 16 + 10 =$ **26 cm**.
d. $V = 4 \times 3 \times 5 = 12 \times 5 =$ **60 cm³**. Volume is measured in cubic units.
e. Replace h with 4: $3 \times 4 = 12$, then $5 + 12 = 17$. Four hours cost **£17**: the £5 fixed charge plus £3 for each hour.

Answer to Question 3

a. Add 5 each time: **24, 29**.
b. Subtract 7 each time: **22, 15**.
c. Add 0.5 each time: **3.5, 4**.
d. Add 3 each time: **5, 8**.
e. Sequence (b) goes **down** each time, so it is the decreasing sequence.

Answer to Question 4

a. The difference is 4 and the first term is 4, so the nth term is $4n$.
b. The difference is 3. $3n$ gives 3, 6, 9, 12. We need 2, 5, 8, 11, which is 1 less each time. The nth term is $3n - 1$.
c. The difference is 5. $5n$ gives 5, 10, 15, 20. We need 2 more each time. The nth term is $5n + 2$.
d. $3 \times 20 + 2 = 60 + 2 =$ **62**.

e. $5 \times 12 - 4 = 60 - 4 =$ **56**.

Answer to Question 5

a. $n + 8 = 21$. Subtract 8 from both sides: $n =$ **13**. Check: 13 + 8 = 21.
b. Divide both sides by 3: $a =$ **8**. Check: 3 × 8 = 24.
c. Subtract 5: $2a = 12$. Divide by 2: $a =$ **6**. Check: 2 × 6 + 5 = 17.
d. Add 3: $4b = 24$. Divide by 4: $b =$ **6**. Check: 4 × 6 − 3 = 21.
e. Multiply both sides by 5: $x =$ **45**. Check: 45 ÷ 5 = 9.

Answer to Question 6

a. For example (1, 9), (2, 8), (5, 5) and (7, 3). Any pair of whole numbers that add up to 10 is correct.
b. If $x = 0$ then $y = 12$. Each time x goes up by 1, y goes down by 2: (0, 12), (1, 10), (2, 8), (3, 6), (4, 4), (5, 2), (6, 0).
c. There are **7 pairs**. x cannot be bigger than 6, because then $2x$ would be more than 12 and y would be negative. So $x = 0, 1, 2, \ldots, 6$ gives every possible pair.

Answer to Question 7

a. Replace m with 7: $C = 2 \times 7 + 3 = 14 + 3 =$ **£17**.
b. Replace m with 10: $C = 2 \times 10 + 3 = 20 + 3 =$ **£23**.
c. Set the cost equal to 25: $2m + 3 = 25$. Undo the +3 by subtracting 3: $2m = 22$. Undo the ×2 by dividing by 2: $m =$ **11 miles**. Check: $2 \times 11 + 3 = 25$.
d. $C = 1.5 \times 10 + 5 = 15 + 5 =$ **£20**. Compare with £23 from part (b): the second company is **cheaper by £3** for 10 miles.

Answer to Question 8

a. $C = 5 + 3 \times 4 =$ **£17**.
b. Yes. The costs 8, 11, 14, 17 go up by £3 each time, which is the same each time, so the sequence is **linear**.
c. $5 + 3h = 29$ gives $3h = 24$ and $h = 8$. At 8 hours the cost is £29, the same as the day ticket. So the day ticket is cheaper for **more than 8 hours**.
d. $C = 5 + 3 \times 6 =$ **£23**. The day ticket costs £29, so it is more expensive. She does **not** save any money: the hourly rate is £6 cheaper.

Answer to Question 9

a. The sequence goes up by 3: 4, 7, 10, **13**, **16**.

b. The difference is 3, so start with $3n$ (3, 6, 9, ...). We need 4, 7, 10, which is 1 more. The nth term is $3n + 1$.
c. $3 \times 20 + 1 =$ **61 sticks**.
d. $3n + 1 \leq 100$ means $3n \leq 99$, so $n \leq 33$. With 33 squares she uses 100 sticks exactly. She can make **33 squares**.

Answer to Question 10

a. $S = 12 + 5 \times 8 = 12 + 40 =$ **£52**.
b. 17, 22, 27, 32.
c. $12 + 5w = 100$ gives $5w = 88$ and $w = 17.6$. Ella cannot save for part of a week, so she needs **18 weeks**.
d. After 6 weeks she has $12 + 30 = 42$, plus £20 gives £62. She needs £38 more: $38 \div 5 = 7.6$, so **8 more weeks**, which is 14 weeks altogether.

Answer to Question 11

a. 12, 14, 16, 18, 20.
b. The difference is 2, so start with $2n$ (2, 4, 6, ...). We need 12, 14, 16, which is 10 more. The nth term is $2n + 10$.
c. $2 \times 15 + 10 =$ **40 seats**.
d. $2n + 10 = 36$ gives $2n = 26$ and $n = 13$. It is **row 13**.
e. $2n + 10 = 45$ gives $2n = 35$ and $n = 17.5$. The row number must be a whole number, so **no**. (All the rows have an even number of seats.)

Answer to Question 12

a. $C = 8 + 0.05 \times 120 = 8 + 6 =$ **£14**.
b. $C = 8 + 0.05 \times 0 =$ **£8**.
c. $8 + 0.05m = 20$ gives $0.05m = 12$ and $m = 240$. He can use **240 minutes**.
d. The second plan is $12 + 0.02m$. Set the two plans equal: $8 + 0.05m = 12 + 0.02m$. Subtract $0.02m$ and 8 from both sides: $0.03m = 4$, so $m = 133.3\ldots$. The plans cost the same at about **133 minutes** (a little over 133).

Answer to Question 13

a. $20 + 6 \times 4 =$ **44 cm**.
b. $20 + 6 \times 10 =$ **80 cm**.
c. $20 + 6 \times 30 = 200$ cm = 2 m. Leo is **right**.
d. $20 + 6w > 100$ gives $6w > 80$ and $w > 13.3$. After 13 weeks it is $20 + 78 = 98$ cm. After **14 weeks** it is 104 cm, which is the first time it is taller than 1 m.

Answer to Question 14

a. Replace g with 15: $6 \times 15 = 90$, then $40 + 90 =$ **£130**.
b. Set up $40 + 6g = 160$. Subtract 40 from both sides: $6g = 120$. Divide by 6: $g =$ **20 guests**. Check: $40 + 6 \times 20 = 160$.
c. Only the fixed hall charge changes, from 40 to 55; the cost per guest stays £6. The new formula is $C = 55 + 6g$.
d. $C = 55 + 6 \times 15 = 55 + 90 =$ **£145**. Difference: $145 - 130 =$ **£15**, which is exactly the rise in the hall price.

Answer to Question 15

a. The output is $3n - 4$.
b. $3 \times 5 - 4 = 15 - 4 = 11$. This matches the table.
c. $3n - 4 = 23$ gives $3n = 27$ and $n =$ **9**. This matches the table.
d. The output is $2(n + 7) = 2n + 14$.
e. The outputs are equal when $3n - 4 = 2n + 14$. Subtract $2n$ from both sides: $n - 4 = 14$. Add 4: $n =$ **18**. Check: $3 \times 18 - 4 = 50$ and $2 \times 25 = 50$.

Answer to Question 16

a. $2 \times 12 + 2w = 40$, so $24 + 2w = 40$ and $2w = 16$, so $w =$ **8 m**.
b. $A = 12 \times 8 =$ **96 m²**.
c. $40 \times 6.50 =$ **£260**.
d. New width = 10 m. New perimeter = $2 \times 12 + 2 \times 10 =$ **44 m**. New area = $12 \times 10 =$ **120 m²**.
e. 120 − 96 = **24 m²**.

Answer to Question 17

a. The sequence adds 3 each time: **14 tiles**.
b. The difference is 3, so start with $3n$ (3, 6, 9). We need 5, 8, 11, which is 2 more. The nth term is $3n + 2$.
c. $3 \times 12 + 2 =$ **38 tiles**.
d. $3n + 2 = 50$ gives $3n = 48$ and $n =$ **16**. It is pattern 16.
e. Pattern 20 uses $3 \times 20 + 2 = 62$ tiles. $62 \times 15\text{p} = 930\text{p} =$ **£9.30**.

Answer to Question 18

a. If $a = 3b$ then $a + b = 3b + b = 4b = 20$, so $b = 5$ and $a =$ **15**. Check: 15 + 5 = 20.
b. Son = 1 part, Mum = 4 parts, so 5 parts = 45. One part = 9. The son is **9** and

Mum is **36**.

c. If the numbers are x and y with $x + y = 15$ and $x - y = 3$, then $2x = 18$, so $x = 9$ and $y =$ **6**. The numbers are **9 and 6**.

d. Three pencils cost $3 \times 0.40 = 1.20$. The two rubbers cost $2.40 - 1.20 = 1.20$. One rubber costs **60p**.

Answer to Question 19

a. $C_A = 5 + 2d$.

b. $C_B = 11 + d$.

c. Alpha: 9, 13, 17, 21. Beta: 13, 15, 17, 19.

d. $5 + 2d = 11 + d$. Subtract d from both sides: $5 + d = 11$. So $d =$ **6 GB**. At 6 GB both cost £17.

e. Alpha: $5 + 2 \times 9 = 23$. Beta: $11 + 9 = 20$. **Beta** is cheaper, by £3 a month. Over a year, he saves $3 \times 12 =$ **£36**.

Answer to Question 20

a. Replace n with 1, 2, 3, 4: 2, 8, 14, 20.

b. $6n - 4 = 62$ gives $6n = 66$ and $n =$ **11**. It is the 11th term.

c. $6n - 4 = 100$ gives $6n = 104$ and $n = 17.33\ldots$. This is not a whole number, so **100 is not a term**. (All the terms are 4 less than a multiple of 6, which means they are all 2 more than a multiple of 6, and 100 is 4 more than a multiple of 6.)

d. $6n - 4 > 200$ gives $6n > 204$ and $n > 34$. For $n = 34$, $6 \times 34 - 4 = 200$, which is not bigger than 200. For $n = 35$: $6 \times 35 - 4 =$ **206**.