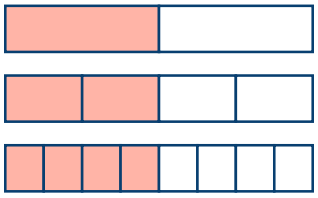
Lesson - Fractions: equivalent, mixed numbers and improper

Year: Year 5

Hello, I am Otto! Fractions describe parts of a whole. You already know halves, quarters and thirds. In Year 5 you discover that the same amount can be written as many different fractions, that fractions can be bigger than one, and how to compare and order them. These skills are used in cooking, measuring and sharing.

Starter activity

Fold a strip of paper in half, then fold it in half again and again. Shade one half. How many quarters are shaded? How many eighths? You should find that 1 half, 2 quarters and 4 eighths are all the same amount of paper. That idea is called equivalence.



All three bars show the same shaded amount: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

Equivalent fractions

Definition: equivalent fractions

Equivalent fractions are fractions that have the same value, even though they look different. $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent. The top number of a fraction is the **numerator** and the bottom number is the **denominator**.

Rule: making equivalent fractions

To make an equivalent fraction, **multiply** the numerator and the denominator by the **same** number. You can also **divide** both by the same number. You must do the same to the top and the bottom, so you are really multiplying by 1.

Example 1

Find the missing number: $\frac{2}{3} = \frac{?}{12}$.

The denominator has been multiplied by 4, because 3 × 4 = 12. Multiply the numerator by 4 too: 2 × 4 = 8. So $\frac{2}{3} = \frac{8}{12}$.

Simplifying fractions

Definition: simplest form

A fraction is in its **simplest form** when the numerator and the denominator have no common factor except 1. We say the fraction is **simplified** or **cancelled down**. $\frac{3}{4}$ is in its simplest form, but $\frac{6}{8}$ is not.

Method: simplifying a fraction

1. Find a common factor of the numerator and the denominator. Look for the highest one, or use a small one such as 2, 3 or 5.
2. Divide both numbers by that factor.
3. Repeat until no common factor is left.

Example 2

Simplify $\frac{24}{36}$.

Both 24 and 36 are divisible by 12, because 24 = 2 × 12 and 36 = 3 × 12. Divide top and bottom by 12 to get $\frac{2}{3}$. The numbers 2 and 3 have no common factor, so the simplest form is $\frac{2}{3}$.

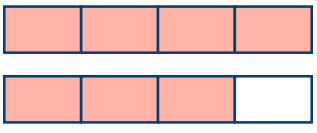
Watch out

Never add or subtract the same number to the top and the bottom. $\frac{1}{2}$ is NOT equal to $\frac{2}{3}$ even though both numbers go up by 1. Only multiplication and division keep the value the same.

Improper fractions and mixed numbers

Definition: improper fraction and mixed number

An **improper fraction** has a numerator that is bigger than or equal to its denominator, such as $\frac{7}{4}$. A **mixed number** has a whole number part and a fraction part, such as $1\frac{3}{4}$. They show the same amount.



Here 4 quarters make one whole bar and 3 more quarters make part of the next. Together that is $\frac{7}{4} = 1\frac{3}{4}$.

Method: improper fraction to mixed number

1. Divide the numerator by the denominator.
2. The whole number part is the quotient (how many times it goes in).
3. The remainder is the new numerator. Keep the same denominator.

Method: mixed number to improper fraction

1. Multiply the whole number by the denominator.
2. Add the numerator.
3. Write the answer over the same denominator.

Example 3

Write $\frac{17}{5}$ as a mixed number, and $2\frac{3}{4}$ as an improper fraction.

17 ÷ 5 = 3 remainder 2, so $\frac{17}{5} = 3\frac{2}{5}$. For $2\frac{3}{4}$: 2 × 4 = 8 and 8 + 3 = 11, so it is $\frac{11}{4}$.

Comparing and ordering fractions

Method: comparing fractions

1. Look at the denominators. If one is a multiple of the other (or both are multiples of a number), change them to the same denominator.
2. Make equivalent fractions with that common denominator.
3. Compare the numerators: the bigger numerator is the bigger fraction.
4. Use the symbols < and > to write the answer.

Example 4

Which is bigger, $\frac{3}{4}$ or $\frac{5}{8}$?

Change $\frac{3}{4}$ to eighths: 3 × 2 = 6, 4 × 2 = 8, so $\frac{3}{4} = \frac{6}{8}$. Now compare $\frac{6}{8}$ with $\frac{5}{8}$. 6 is more than 5, so $\frac{3}{4} > \frac{5}{8}$. The bigger fraction is $\frac{3}{4}$.

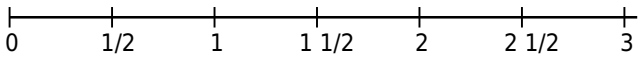
Example 5

Put $\frac{5}{6}$, $\frac{2}{3}$ and $\frac{7}{12}$ in order, smallest first.

The common denominator is 12 because 6 and 3 both go into 12. $\frac{5}{6} = \frac{10}{12}$, $\frac{2}{3} = \frac{8}{12}$ and $\frac{7}{12}$ stays the same. The order of numerators is 7, 8, 10. So the order is $\frac{7}{12}$, $\frac{2}{3}$, $\frac{5}{6}$.

Key fact: fractions on a number line

Fractions and mixed numbers sit on a number line between the whole numbers. $1\frac{1}{2}$ is halfway between 1 and 2. An improper fraction such as $\frac{7}{4}$ is just past 1 and a half, and is the same point as $1\frac{3}{4}$. The number line is a good way to compare.



Did you know?

The Egyptians wrote fractions about 4,000 years ago, but they only used fractions with 1 on top (unit fractions) such as one half or one fifth. To write $\frac{3}{4}$ they wrote one half plus one quarter. The line we use between the numerator and the denominator was first used by Arab mathematicians around 1,000 years ago.

Otto's tip

When you compare fractions, always ask: can I make the denominators the same? If you do that first, the comparison is easy because you only compare the numerators.