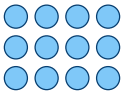
Lesson - Factors, multiples, primes, squares and cubes

Year: Year 5

Hello, I am Otto! Numbers have secrets. Some numbers can be shared out equally in many ways, like 24 sweets. Others, like 13, can only be shared one way. In this chapter you will meet factors, multiples, prime numbers, composite numbers, square numbers and cube numbers. These ideas help with fractions, with times tables and with solving puzzles.

Starter activity

Make 12 counters into a rectangle. You can make a rectangle with 1 row of 12, 2 rows of 6 or 3 rows of 4. Now try 13 counters. You can only make 1 row of 13. Keep thinking about why 12 and 13 behave so differently as you read on.



This array has 3 rows of 4 dots, so it shows 3 × 4 = 12.

Factors and factor pairs

Definition: factor

A **factor** of a number is a whole number that divides it exactly, leaving no remainder. 6 is a factor of 24 because 24 ÷ 6 = 4. Every number has 1 and itself as factors.

Method: listing all the factors

1. Start with the pair 1 and the number itself.
2. Test 2, then 3, then 4 and so on. When a number divides exactly, write down the pair, for example 3 and 8 for 24.
3. Stop when you reach a factor that you have already written down as the partner of a smaller one.
4. Write the factors in order, smallest first.

Example 1

List all the factors of 36 and write its factor pairs.

1 × 36, 2 × 18, 3 × 12, 4 × 9 and 6 × 6. The factors are **1, 2, 3, 4, 6, 9, 12, 18, 36**. The pair 6 × 6 gives the factor 6 only once, which is why 36 has an odd number of factors.

Key fact: divisibility tests

A number is divisible by 2 if it ends in 0, 2, 4, 6 or 8. It is divisible by 5 if it ends in 0 or 5, and by 10 if it ends in 0. It is divisible by 3 if its digits add to a multiple of 3, and by 9 if the digits add to a multiple of 9. For example 4,572 has digit sum 4 + 5 + 7 + 2 = 18, so it is divisible by both 3 and 9.

Multiples and common multiples

Definition: multiple

A **multiple** of a number is a number in its times table. The multiples of 7 are 7, 14, 21, 28, 35 and so on. A multiple is made by multiplying the number by 1, 2, 3, 4 and so on.

Definition: common factor and common multiple

A **common factor** of two numbers divides them both. A **common multiple** is in both times tables. The **highest common factor** is the biggest common factor and the **lowest common multiple** is the smallest common multiple.

Example 2

Find the lowest common multiple of 6 and 8, and the highest common factor of 18 and 30.

Multiples of 6: 6, 12, 18, 24, 30. Multiples of 8: 8, 16, 24, 32. The first number in both lists is 24, so the lowest common multiple is **24**. Factors of 18: 1, 2, 3, 6, 9, 18. Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30. The common factors are 1, 2, 3 and 6, so the highest common factor is **6**.

Watch out

Factors are never bigger than the number, but multiples are never smaller. Many pupils mix up the words. Remember: factors are found by dividing and are few in number, multiples are found by multiplying and go on for ever.

Prime and composite numbers

Definition: prime number

A **prime number** has exactly two factors, 1 and itself. The prime numbers up to 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. A number with more than two factors is called a **composite number**. The number 1 is neither prime nor composite, because it has only one factor.

Key fact: facts about primes

2 is the only even prime number. Every other even number has the factor 2. No prime number bigger than 5 ends in 0, 2, 4, 5, 6 or 8. To test if a number up to 100 is prime, try dividing it by 2, 3, 5 and 7. If none of them works, the number is prime. Beware: 91 = 7 × 13 is composite even though it looks like a prime.

Definition: prime factor

A **prime factor** is a factor that is also a prime number. Every composite number can be written as a product of its prime factors in exactly one way, apart from the order. For example 60 = 2 × 2 × 3 × 5.

Method: a factor tree

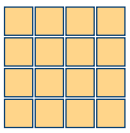
1. Write the number and split it into any two factors.
2. Circle any factor that is prime and split any factor that is not.
3. Keep going until every branch ends in a prime.
4. Write the primes in order as a product. Use powers if you like: 60 = 2^2 × 3 × 5.

Example 3

Write 72 as a product of prime factors.

72 = 8 × 9. Then 8 = 2 × 2 × 2 and 9 = 3 × 3. So 72 = 2 × 2 × 2 × 3 × 3 = **2^3 × 3^2**.

Square numbers and cube numbers



A 4 by 4 array of tiles has 16 tiles. 16 is a square number.

Definition: square number

A **square number** is a number multiplied by itself. We write it with a small 2: 4^2 = 4 × 4 = 16. The first twelve squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121 and 144. A square number has an odd number of factors.

Definition: cube number

A **cube number** is a number multiplied by itself and then by itself again. We write it with a small 3: 3^3 = 3 × 3 × 3 = 27. The first five cubes are 1, 8, 27, 64 and 125, and 10^3 = 1,000.

Example 4

Work out 5^2 and 5^3, and say whether 50 is a square number.

$5^2 = 5 \times 5 = \mathbf{25}$. $5^3 = 5 \times 5 \times 5 = 25 \times 5 = \mathbf{125}$. For 50: $7^2 = 49$ and $8^2 = 64$, so 50 lies between two squares and is **not a square number**.

Watch out

5^2 does NOT mean 5×2. It means 5×5. In the same way 5^3 is not 5×3. Check by working out $2^2 = 4$ and $2 \times 2 = 4$, but $3^2 = 9$ while $3 \times 2 = 6$.

Example 5

A hall has 144 chairs in a square arrangement. How many chairs are in each row?

The rows and columns are equal, so the number in each row is the square root of 144. Since $12 \times 12 = 144$, there are **12 chairs in each row**.

Did you know?

The ancient Greeks called numbers like 1, 4, 9 and 16 "square" because they thought of them as dots in a square pattern. About 2,300 years ago a Greek mathematician called Eratosthenes invented a clever way to find primes. You write the numbers on a grid and cross out multiples of 2, 3, 5 and 7. The numbers left are primes. It is called the Sieve of Eratosthenes.

Otto's tip

To check whether a number is prime, divide it by 2, 3, 5 and 7 only. For numbers under 100, if none of them divide exactly, the number is prime. Learn the primes up to 20 by heart, they appear in lots of questions!