Lesson - Equivalent fractions and comparing fractions

Year: Year 3

Hello, I am Otto! Fractions are everywhere: half a pizza, a quarter of an hour, three tenths of a bar of chocolate. In this chapter you learn that the same amount can be written as different fractions, and you learn how to compare fractions to decide which is bigger.

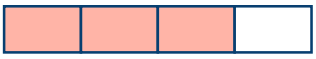
Starter activity

Take two identical strips of paper. Fold the first strip in half and colour one half. Fold the second strip into four equal parts and colour two of the parts. Hold the strips side by side. They show the same amount! One half and two quarters are equal. You have discovered equivalent fractions with your own hands.

Fractions: a reminder

Definition: fraction

A **fraction** describes equal parts of a whole. The bottom number is the **denominator**: it tells you how many equal parts the whole is cut into. The top number is the **numerator**: it tells you how many parts you have. In $\frac{3}{4}$ the whole is cut into 4 equal parts and we have 3 of them.



Three of the four equal parts are shaded: $\frac{3}{4}$ of the bar.

Example 1

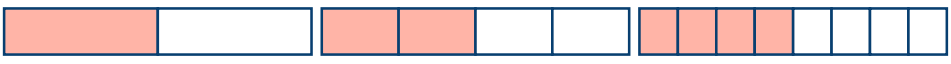
A pizza is cut into 8 equal slices and Sam eats 5. What fraction of the pizza does he eat?

The denominator is 8 because there are 8 equal slices. The numerator is 5 because he eats 5. The fraction is $\frac{5}{8}$.

Equivalent fractions

Definition: equivalent fractions

Equivalent fractions are different fractions that show the same amount. For example $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ are all equivalent, because each one is half of the whole.



The three bars are the same length. The shaded part is the same size in each bar, so $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

Rule: making equivalent fractions

Multiply the numerator and the denominator by the **same number** to get an equivalent fraction. For example, $\frac{1}{3} = \frac{2}{6}$ because $1 \times 2 = 2$ and $3 \times 2 = 6$. You can also divide both by the same number: $\frac{4}{8} = \frac{1}{2}$ because $4 \div 4 = 1$ and $8 \div 4 = 2$.

Example 2

Write a fraction equivalent to $\frac{2}{5}$ with denominator 10.

The denominator goes from 5 to 10, so we multiply by 2. Multiply the numerator by 2 as well: $2 \times 2 = 4$. So $\frac{2}{5} = \frac{4}{10}$.

Method: finding the missing number in equivalent fractions

1. Look at the two denominators. What do you multiply (or divide) to get from one to the other?
2. Do the same to the numerator.
3. Check with a picture or a fraction wall.

Example 3

Fill in the gap: $\frac{3}{4} = \frac{?}{12}$

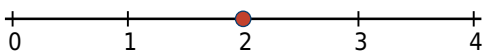
$4 \times 3 = 12$, so multiply the numerator by 3 as well: $3 \times 3 = 9$. The answer is $\frac{3}{4} = \frac{9}{12}$.

Watch out

Do not add the same number to the top and bottom. $\frac{1}{2}$ is not equal to $\frac{2}{3}$. We must **multiply** or **divide** the top and bottom by the same number. Another common slip is to think a bigger denominator means a bigger fraction. It actually means smaller pieces.

Fractions on a number line

A number line from 0 to 1 can be split into equal parts to show fractions. If we split the line into 4 equal parts, the marks are at $\frac{0}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$. The mark $\frac{2}{4}$ is in the same place as $\frac{1}{2}$, which shows that they are equivalent. The mark $\frac{4}{4}$ is the same as 1 whole.



Counting quarters: 0, one quarter, two quarters (one half), three quarters, one whole.

Comparing fractions with the same denominator

Rule: same denominator

When two fractions have the **same denominator**, the one with the bigger numerator is bigger. The parts are the same size, so more parts means more. For example $\frac{5}{8} > \frac{3}{8}$.

Example 4

Which is greater, $\frac{4}{9}$ or $\frac{7}{9}$?

The denominators are both 9, so compare the numerators: 7 is more than 4. So $\frac{7}{9} > \frac{4}{9}$. The answer is $\frac{7}{9}$.

Comparing unit fractions

Rule: unit fractions

A **unit fraction** has 1 as its numerator, such as $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{10}$. When the numerators are the same, the fraction with the **smaller denominator** is bigger. Cutting a cake into 3 pieces gives bigger pieces than cutting it into 10 pieces. So $\frac{1}{3} > \frac{1}{10}$.

Example 5

Put $\frac{1}{5}$, $\frac{1}{2}$ and $\frac{1}{8}$ in order, smallest first.

The denominators are 5, 2 and 8. The biggest denominator gives the smallest fraction. The order is $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{2}$.

Comparing using equivalent fractions

Method: comparing fractions with different denominators

1. Look for a denominator you can reach by multiplying both fractions (for halves, quarters and eighths, 8 works).
2. Write each fraction with that same denominator.
3. Compare the numerators.
4. Write the answer using the original fractions.

Example 6

Which is bigger, $\frac{3}{4}$ or $\frac{5}{8}$?

Change $\frac{3}{4}$ to eighths: $\frac{3}{4} = \frac{6}{8}$. Now compare $\frac{6}{8}$ and $\frac{5}{8}$. 6 is more than 5, so $\frac{3}{4} > \frac{5}{8}$. The

bigger fraction is $\frac{3}{4}$.

Adding and subtracting fractions with the same denominator

Rule: adding and subtracting

When the denominators are the same, add or subtract only the numerators. The denominator stays the same, because the size of the pieces does not change. $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$ and $\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$.

Example 7

Work out $\frac{5}{10} + \frac{3}{10}$ and $\frac{1}{2} = \frac{?}{10}$.

5 tenths and 3 tenths make 8 tenths: $\frac{8}{10}$. For the second part, 2 × 5 = 10, so multiply 1 by 5: $\frac{1}{2} = \frac{5}{10}$.

Did you know?

The Ancient Egyptians wrote nearly every fraction as a sum of unit fractions. They had a special sign for one third and another for two thirds. A fraction wall, with rows for halves, thirds, quarters and so on, is a modern way to see equivalent fractions at a glance.

Otto's tip

Draw a fraction bar when you get stuck. Draw two bars of the same length, split them into the two denominators, shade and compare. Seeing the pieces makes the answer clear.