Test answers - Vectors in three dimensions

Year: Year 13 · Duration: 40 minutes · /20

Answer to Question 1 (4 marks)

Method: work component by component. For $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the magnitude is $|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$, and the vector from A to B is [illegible]. Mark scheme as shown in brackets.

a. Add the components. $2\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$ (1).
b. Multiply each vector by its scalar first, then subtract component by component. $(9 + 2)\mathbf{i} + (-6 - 8)\mathbf{j} + (15 - 4)\mathbf{k} = 11\mathbf{i} - 14\mathbf{j} + 11\mathbf{k}$ (2).
c. Square the components, add and take the square root. $\sqrt{9 + 4 + 25} = \sqrt{38} = 6.16$ (1). So $|\mathbf{a}| = \sqrt{38} \approx 6.16$.

Answer to Question 2 (4 marks)

Method: work component by component. For $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the magnitude is $|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$, and the vector from A to B is [illegible]. Mark scheme as shown in brackets.

a. Subtract the coordinates of A from those of B. [illegible] (1);
$AB = \sqrt{9 + 16 + 144} = 13$ (1).
b. [illegible] has only a $\mathbf{k}$ component, so AC is its size. [illegible] so $AC = 5$;
$BC = \sqrt{9 + 16 + 49} = \sqrt{74}$ (1). So $BC = \sqrt{74} \approx 8.60$.
c. Use the cosine rule with the three side lengths. $\cos A = \dfrac{169 + 25 - 74}{130} = \dfrac{12}{13}$, $A = 22.6^\circ$ (1). So angle $BAC \approx 22.6^\circ$.

Answer to Question 3 (4 marks)

a. [illegible] (1). [illegible] must be parallel with scale factor 1, so $p - 2 = 3$ and $p = 5$ (1).

b. [illegible] and [illegible] (1), so $AB : BC = 1 : 2$ (1).

Since [illegible] and [illegible] has $\mathbf{i}$-component 2, B is one third of the way from A to C. Check: [illegible] ✓. So AB is half of BC in length.

Answer to Question 4 (4 marks)

Method: when $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, the particle moves with constant velocity $\mathbf{v}$. Its speed is $|\mathbf{v}|$. Mark scheme as shown in brackets.

a. The velocity is the vector of coefficients of t. Velocity $= 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$ m/s (1). Speed $= \sqrt{9 + 4 + 36} = 7$ m/s (1). So the particle moves at a constant 7 m s^{-1}.

b. Substitute $t = 5$ and find the magnitude of the position vector. $\mathbf{r} = 16\mathbf{i} - 8\mathbf{j} + 34\mathbf{k}$ (1). Distance $= \sqrt{256 + 64 + 1156} = \sqrt{1476} = 38.4$ m (1). So after 5 seconds the particle is about 38.4 m from the origin.

Answer to Question 5 (4 marks)

a. $\mathbf{F}_1 + \mathbf{F}_2 = -4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, so $\mathbf{F}_3 = 4\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$ (1).
$|\mathbf{F}_3| = \sqrt{16 + 4 + 36} = \sqrt{56} = 7.48$ N (1).

b. Resultant $= -4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$ and $\mathbf{a} = \frac{1}{2}(-4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}) = -2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ m/s^2 (1).
$|\mathbf{a}| = \sqrt{4 + 1 + 9} = \sqrt{14} = 3.74$ m/s^2 (1).

In equilibrium the resultant of all three forces is zero, so $\mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2)$. Once $\mathbf{F}_3$ is removed, the resultant is $-\mathbf{F}_3$ and $\mathbf{F} = m\mathbf{a}$ gives the acceleration. Check: $|\mathbf{a}| = \frac{\sqrt{56}}{2} = \sqrt{14}$ ✓.