Test - Proof by contradiction

Year: Year 13 · Duration: 30 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Parity of squares

(a) Write the negation of the statement "n is an even integer". [1 mark] (b) Prove by contradiction that if n^2 is even, where n is an integer, then n is even. [3 marks]

Question 2 (4 marks)

Question 2 – The square root of five

Prove by contradiction that $\sqrt{5}$ is irrational. [4 marks]

Question 3 (4 marks)

Question 3 – No integer solutions

Prove by contradiction that there are no integers m and n such that $12m + 18n = 5$. [4 marks]

Question 4 (4 marks)

Question 4 – Infinitely many primes

Complete Euclid’s proof that there are infinitely many primes. (a) Assume that there are finitely many primes $p_1, p_2, \ldots, p_k$. Define a number N which gives a contradiction. [1 mark] (b) Show that N is not divisible by any p_i. [2 marks] (c) State the contradiction. [1 mark]

Question 5 (4 marks)

Question 5 – Irrational products

(a) Prove by contradiction that if x is a non-zero rational number and y is irrational,

then xy is irrational. [3 marks] (b) Hence state, with a reason, whether $\frac{\sqrt{2}}{3}$ is rational. [1 mark]