Multiple choice questions - Proof by contradiction

Year: Year 13 · 10

Question 1 What is the first line of a proof by contradiction that $\sqrt{5}$ is irrational?

- ◯ **A.** Assume $\sqrt{5}$ is rational
- ◯ **B.** Assume $\sqrt{5}$ is irrational
- ◯ **C.** Assume $\sqrt{5} = 2.236$
- ◯ **D.** Assume $\sqrt{5}$ is an integer

Question 2 The negation of "all prime numbers are odd" is

- ◯ **A.** no prime number is odd
- ◯ **B.** some prime number is even
- ◯ **C.** all prime numbers are even
- ◯ **D.** some prime number is odd

Question 3 In the proof that $\sqrt{2}$ is irrational, the contradiction is that

- ◯ **A.** a is even and b is odd
- ◯ **B.** a squared is odd
- ◯ **C.** a and b are both even but the fraction was in lowest terms
- ◯ **D.** b is zero

Question 4 Which statement is a valid counterexample to "if n is prime then $n + 2$ is prime"?

- ◯ **A.** $n = 3$
- ◯ **B.** $n = 5$
- ◯ **C.** $n = 11$
- ◯ **D.** $n = 7$

Question 5 If $N = 2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031$, then

- ◯ **A.** N is not prime, but has prime factors not in the list
- ◯ **B.** N is prime
- ◯ **C.** N is divisible by 2
- ◯ **D.** N is divisible by 13

Question 6 Which of these cannot be proved by giving examples?

- ◯ **A.** n^2 for some integer n

- ◯ **B.** $n^2 \geq n$ for all integers n
- ◯ **C.** $2^n > n^2$ is false for some n
- ◯ **D.** there exists an even prime

Question 7 If x is irrational, then $x + \frac{1}{2}$ is

- ◯ **A.** rational
- ◯ **B.** an integer
- ◯ **C.** irrational
- ◯ **D.** could be either

Question 8 Which is the correct negation of "n^2 is even"?

- ◯ **A.** n is odd
- ◯ **B.** n^2 is a multiple of 3
- ◯ **C.** n^2 is prime
- ◯ **D.** n^2 is odd

Question 9 Prove $14m + 21n = 1$ has no integer solutions: the key fact is that

- ◯ **A.** the left side is a multiple of 7
- ◯ **B.** the left side is even
- ◯ **C.** m and n must be positive
- ◯ **D.** the left side is prime

Question 10 In the proof of infinitely many primes, which fact leads to the contradiction?

- ◯ **A.** N is always prime
- ◯ **B.** a prime dividing N and $p_1 \cdots p_k$ divides 1
- ◯ **C.** N is even
- ◯ **D.** $p_1 \cdots p_k$ is odd

Answer grid - Proof by contradiction

Year: Year 13

Question	Answer	Explanation
1	**A**	You assume the opposite of what you want to prove: that $\sqrt{5}$ is rational.
2	**B**	Negating "for all" gives "there exists" with the property reversed.
3	**C**	Both a and b have factor 2.
4	**D**	7 is prime but 9 is not. For 3, 5 and 11, n+2 is 5, 7 and 13.
5	**A**	30031 = 59 x 509; neither of these is in the list.
6	**B**	Examples cannot prove a "for all" claim.
7	**C**	Rational plus irrational is irrational.
8	**D**	The opposite of even is odd (for the same quantity, n^2).
9	**A**	7(2m+3n) cannot equal 1.
10	**B**	The difference N minus the product is 1.