Test answers - Projectiles and variable acceleration

Year: Year 13 · Duration: 40 minutes · /20

Answer to Question 1 (4 marks)

Method: treat the horizontal and vertical motions separately. Horizontally the velocity $u\cos\theta$ is constant; vertically the acceleration is $g = 9.8$ m s^{-2} downwards, so use the suvat equations with initial velocity $u\sin\theta$. The time t links the two motions. The ball is thrown horizontally, so the initial vertical velocity is zero. Mark scheme as shown in brackets.

a. $40 = 4.9t^2$, $t = 2.86$ s (1). $x = 15 \times 2.857 = 42.9$ m (1).

b. $v_y = 9.8 \times 2.857 = 28.0$, speed $= \sqrt{225 + 784} = 31.8$ m s^{-1} (1) at $\tan^{-1}\dfrac{28}{15} = 61.8^\circ$ below the horizontal (1).

Answer: the ball hits the sea after about 2.86 s, 42.9 m from the cliff, at about 31.8 m s^{-1} and 61.8° below the horizontal.

Answer to Question 2 (4 marks)

Method: treat the horizontal and vertical motions separately. Horizontally the velocity $u\cos\theta$ is constant; vertically the acceleration is $g = 9.8$ m s^{-2} downwards, so use the suvat equations with initial velocity $u\sin\theta$. The time t links the two motions. Mark scheme as shown in brackets.

a. The ball returns to the ground when the vertical displacement is zero, at $t = \dfrac{2u\sin\theta}{g}$. $T = \dfrac{2 \times 26\sin 35^\circ}{9.8} = 3.04$ s (2). So the ball is in the air for about 3.04 s.

b. Use $R = \dfrac{u^2\sin 2\theta}{g}$ and $H = \dfrac{(u\sin\theta)^2}{2g}$. $R = \dfrac{26^2\sin 70^\circ}{9.8} = 64.8$ m (1); $H = \dfrac{(26\sin 35^\circ)^2}{19.6} = 11.3$ m (1). So the range is about 64.8 m and the greatest height about 11.3 m.

Answer to Question 3 (4 marks)

Method: treat the horizontal and vertical motions separately. Horizontally the velocity $u\cos\theta$ is constant; vertically the acceleration is $g = 9.8$ m s^{-2} downwards, so use the suvat equations with initial velocity $u\sin\theta$. The time t links the two motions. Mark scheme as shown in brackets.

a. Write x and y in terms of t, make t the subject of the x equation and substitute. $x = 20\cos 30^\circ\, t$, $y = 20\sin 30^\circ\, t - 4.9t^2$ (1). Eliminate $t = \dfrac{x}{20\cos 30^\circ}$ to get the result (1). The path is a parabola.

b. Substitute $x = 25$, $\tan 30^\circ = 0.5774$ and $\cos^2 30^\circ = 0.75$.
$y = 25\tan 30^\circ - \dfrac{9.8 \times 625}{800 \times 0.75} = 14.434 - 10.208 = 4.23$ m (2). So the ball is about 4.23 m high when it has travelled 25 m horizontally.

Answer to Question 4 (4 marks)

Method: displacement is $\int v\,dt$, but distance travelled must count every part of the motion as positive. Find when $v = 0$, integrate separately on each interval where v keeps the same sign, and add the sizes of the results. Mark scheme as shown in brackets.

a. $(t-1)(t-5) = 0$, so at rest at $t = 1$ and $t = 5$ (1).

b. $\left[\frac{t^3}{3} - 3t^2 + 5t\right]_0^5 = \frac{125}{3} - 75 + 25 = -\frac{25}{3} = -8.33$ m (1).

c. From 0 to 1: $\frac{1}{3} - 3 + 5 = \frac{7}{3}$ m forwards (1). From 1 to 5: $-\frac{25}{3} - \frac{7}{3} = -\frac{32}{3}$, i.e. $\frac{32}{3}$ m backwards. Total distance $= \frac{7}{3} + \frac{32}{3} = 13$ m (1).

Answer to Question 5 (4 marks)

Method: when the acceleration depends on t, suvat cannot be used. Differentiate to go from displacement to velocity to acceleration, and integrate (adding a constant found from the initial conditions) to go back. Mark scheme as shown in brackets.

a. $\mathbf{v} = 4t\mathbf{i} + (3t^2 - 3)\mathbf{j}$, $\mathbf{a} = 4\mathbf{i} + 6t\mathbf{j}$ (2).

b. At $t = 2$: $\mathbf{v} = 8\mathbf{i} + 9\mathbf{j}$, speed $= \sqrt{145} = 12.0$ m s^{-1} (1). $\mathbf{a} = 4\mathbf{i} + 12\mathbf{j}$, $|\mathbf{a}| = \sqrt{160} = 12.6$ m s^{-2} (1).