Worksheet - Projectiles and variable acceleration

Year: Year 13 · 21

Show your working.

Question 1

Question 1 – Components of velocity *(Section A · Skills practice)*

A ball is projected from level ground with speed 24 m s^{-1} at 40° above the horizontal. Take $g = 9.8$ m s^{-2}.

a. Find the initial horizontal and vertical components of the velocity. [2 marks]
b. Find the time taken to reach the highest point. [2 marks]
c. Find the greatest height reached. [2 marks]

Question 2

Question 2 – Thrown horizontally *(Section A · Skills practice)*

A ball rolls off the edge of a horizontal table of height 1.2 m with speed 3 m s^{-1}.

a. Find the time taken to reach the floor. [2 marks]
b. Find the horizontal distance from the table edge to the landing point. [1 mark]
c. Find the speed and direction of the ball as it lands. [4 marks]

Question 3

Question 3 – Time of flight and range *(Section A · Skills practice)*

A golf ball is hit from level ground with speed 30 m s^{-1} at 25° above the horizontal.

a. Find the time of flight. [2 marks]
b. Find the range. [2 marks]
c. Find the greatest height. [2 marks]

Question 4

Question 4 – From acceleration to displacement *(Section A · Skills practice)*

A particle moves in a straight line. At time $t = 0$ it is at the origin with velocity 3 m s^{-1}. Its acceleration at time t seconds is $a = 4t - 6$ m s^{-2}.

a. Find an expression for the velocity. [3 marks]
b. Find an expression for the displacement from the origin. [3 marks]

c. Find the velocity and displacement when $t = 2$. [2 marks]

Question 5

Question 5 – Differentiating velocity *(Section A · Skills practice)*

A particle moves in a straight line with velocity $v = 3t^2 - 12t + 9$ m s^{-1} for $t \geq 0$.

a. Find the acceleration at time t. [2 marks]
b. Find the times at which the particle is instantaneously at rest. [2 marks]
c. Find the acceleration at each of those times and describe the motion. [3 marks]

Question 6

Question 6 – Displacement and distance *(Section A · Skills practice)*

A particle moves in a straight line with velocity $v = t^2 - 6t + 8$ m s^{-1} for $0 \leq t \leq 4$.

a. Find the times when $v = 0$. [2 marks]
b. Find the displacement of the particle from $t = 0$ to $t = 4$. [3 marks]
c. Find the total distance travelled in the 4 seconds. [3 marks]

Question 7

Question 7 – The free kick *(Section B · Real-life practice)*

A footballer takes a free kick from level ground 20 m from the goal line. The ball leaves the ground at 18 m s^{-1} at 22° above the horizontal. A defensive wall 2 m high stands 9.15 m from the ball. The crossbar is 2.44 m high. Model the ball as a particle.

a. Find the height of the ball when it is above the wall, and decide if it clears the wall. [4 marks]
b. Find the height of the ball when it reaches the goal line. [3 marks]
c. Does the ball go under the crossbar? [1 mark]
d. State two factors that the model ignores and say how each would affect the answer. [2 marks]

Question 8

Question 8 – The cliff diver *(Section B · Real-life practice)*

A diver runs off a horizontal cliff top of height 25 m with horizontal speed 2 m s^{-1}. Rocks extend 3 m out from the base of the cliff.

a. Find the time taken to reach the water. [2 marks]
b. Show that the diver clears the rocks. [2 marks]
c. Find the speed and angle of entry into the water. [4 marks]

Question 9

Question 9 – A garden fountain *(Section B · Real-life practice)*

Water leaves a nozzle at ground level with speed 8 m s^{-1} at 60° to the horizontal. Treat the water as a particle.

a. Find the maximum height of the jet. [3 marks]
b. Find the horizontal range. [3 marks]
c. The designer wants to double the range at the same angle. By what factor must the speed increase? [2 marks]

Question 10

Question 10 – A cricket catch *(Section B · Real-life practice)*

A cricket ball is hit from a height of 1 m above the ground with speed 15 m s^{-1} at 35° above the horizontal.

a. Find the greatest height of the ball above the ground. [3 marks]
b. Find the time at which the ball reaches the ground. [4 marks]
c. Find the horizontal distance from the batter to the point where the ball would land. [2 marks]

Question 11

Question 11 – A moving lift *(Section B · Real-life practice)*

The velocity of a lift in a tall building is modelled by $v = 0.5t - 0.05t^2$ m s^{-1} for $0 \le t \le 10$ seconds, where t is time since the lift left the ground floor.

a. Show that the lift is at rest at $t = 0$ and $t = 10$. [2 marks]
b. Find the maximum speed. [3 marks]
c. Find the height gained during the 10 seconds. [3 marks]
d. Find the acceleration at $t = 8$ and explain its sign. [2 marks]

Question 12

Question 12 – Jogger on a straight track *(Section B · Real-life practice)*

A jogger runs along a straight track. Her velocity at time t minutes is $v = t^2 - 8t + 12$ m s^{-1} for $0 \le t \le 6$, where positive is towards the park gate.

a. Find the times at which she turns round. [2 marks]
b. Show that her displacement from the start after 6 minutes is zero. [3 marks]
c. Find the total distance run. [3 marks]

Question 13

Question 13 – The train leaving the station *(Section B · Real-life practice)*

A train accelerates from rest at the station. Its acceleration is modelled by $a = 2 - 0.1t$ m s^{-2}, for $0 \le t \le 20$, where t is in seconds.

a. Find an expression for the velocity of the train. [3 marks]
b. Find the speed when the acceleration becomes zero. [2 marks]
c. Find the distance covered in the first 20 s. [3 marks]

Question 14

Question 14 – A drone in a gust *(Section B · Real-life practice)*

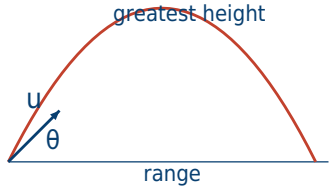
A drone moves so that its position vector relative to a fixed point is $\mathbf{r} = 2t\mathbf{i} + (t^2 - 4t)\mathbf{j}$ metres at time t seconds.

a. Find the velocity and acceleration at time t. [3 marks]
b. Find the speed at $t = 3$. [2 marks]
c. Find the time when the speed is least and the least speed. [3 marks]

Question 15

Question 15 – The free throw *(Section C · Problem solving)*

In a basketball game a player releases the ball at a height of 2.1 m, at 50° above the horizontal. The hoop is at a height of 3.05 m and the centre of the hoop is 4.6 m horizontally from the release point. Model the ball as a particle.



a. Show that the path of the ball can be written $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$, where y is the height above the release point. [4 marks]
b. Find the speed of release that takes the ball through the centre of the hoop. [5 marks]
c. Find the time taken for the ball to reach the hoop. [2 marks]
d. The ball is released 0.1 m higher but with the same speed. Describe qualitatively how the ball reaches the hoop. [1 mark]

Question 16

Question 16 – Range and launch angle *(Section C · Problem solving)*

A sports scientist fires balls from level ground at 25 m s^{-1} at several angles. The theoretical ranges (no air resistance) are in the table.

Angle θ	15°	30°	45°	60°	75°
Range (m)	31.9	55.2	63.8	55.2	31.9

a. Verify the range for $\theta = 30^\circ$. [2 marks]
b. Explain, using $R = \dfrac{u^2 \sin 2\theta}{g}$, why the ranges for 15° and 75° are the same and why 45° is best. [3 marks]
c. Find the two possible angles for a range of 50 m. [4 marks]
d. A real ball launched at 30° travels 52 m. Compare it with the model and give a reason. [2 marks]

Question 17

Question 17 – Two-stage motion with variable acceleration *(Section C · Problem solving)*

A particle moves along a straight line. Its velocity is $v = t^3 - 6t^2 + 9t$ m s^{-1} at time t seconds, for $t \geq 0$. At $t = 0$ it is at the origin O.

a. Show that $v = t(t-3)^2$ and describe when the particle is at rest. [3 marks]
b. Find the displacement at time t. [3 marks]
c. Find the greatest speed for $0 \leq t \leq 3$. [4 marks]
d. Find the acceleration and displacement when $t = 4$. [3 marks]

Question 18

Question 18 – A cliff-top throw *(Section C · Problem solving)*

A stone is thrown from the top of a cliff 30 m above sea level with speed 18 m s^{-1} at 40° above the horizontal.

Quantity	Value
Launch height	30 m
Speed	18 m s^{-1}
Angle	40° above horizontal

a. Find the greatest height above the sea. [3 marks]
b. Find the time taken to hit the sea. [4 marks]
c. Find the horizontal distance from the foot of the cliff to the landing point. [2 marks]

d. Find the speed and the direction of motion when the stone hits the sea. [4 marks]

Question 19

Question 19 – From the path to the launch conditions *(Section C · Problem solving)*

The path of a jet of water from a sprinkler is observed to be $y = x - 0.05x^2$ for $y \geq 0$, where x and y are measured in metres from the nozzle.

a. Find the angle of projection. [2 marks]

b. Use the equation $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$ to find the speed of projection. [4 marks]

c. Find the range and the greatest height of the jet. [4 marks]

d. The gardener wants to wet a flower 15 m away at a height of 1 m. Will the jet pass through that point? [2 marks]

Question 20

Question 20 – Challenge: height and range *(Section D · Challenge)*

A particle is projected from level ground with speed u at angle θ above the horizontal. The range is R and the greatest height is H.

a. Show that $H = \dfrac{R\tan\theta}{4}$. [4 marks]

b. Check the result when $u = 20$ and $\theta = 30^\circ$. [2 marks]

Question 21

Question 21 – Challenge: resisted motion *(Section D · Challenge)*

A particle moves on a smooth horizontal surface in a liquid so that its acceleration is $a = -0.5v$ m s^{-2}. At $t = 0$ its velocity is 20 m s^{-1}.

a. By writing $a = \dfrac{dv}{dt}$ and separating variables, show that $v = 20e^{-0.5t}$. [4 marks]

b. Find the time taken for the speed to halve. [2 marks]

c. By integrating, show that the particle never travels more than 40 m. [4 marks]