Test answers - Parametric equations

Year: Year 13 · Duration: 35 minutes · /20

Answer to Question 1 (4 marks)

Method: to find a Cartesian equation, eliminate the parameter. If one equation is linear in t, make t the subject and substitute. If the equations involve $\sin t$ and $\cos t$, isolate them and use $\sin^2 t + \cos^2 t = 1$ or a double angle formula. Mark scheme as shown in brackets.

a. Use the x coordinate to find t, then check with the y coordinate. $3t - 1 = 8$ gives $t = 3$ (1); check $y = 9 + 2 = 11$ (1). So the point $(8, 11)$ corresponds to $t = 3$.

b. Make t the subject of the linear equation and substitute into y. $t = \dfrac{x+1}{3}$ (1), so $y = \dfrac{(x+1)^2}{9} + 2$ (1). The curve is a parabola with vertex $(-1, 2)$.

Answer to Question 2 (4 marks)

Method: to find a Cartesian equation, eliminate the parameter. If one equation is linear in t, make t the subject and substitute. If the equations involve $\sin t$ and $\cos t$, isolate them and use $\sin^2 t + \cos^2 t = 1$ or a double angle formula. Mark scheme as shown in brackets.

$\cos t = \dfrac{x-1}{3}$, $\sin t = \dfrac{y-2}{3}$ (1). $(x-1)^2 + (y-2)^2 = 9$ (2). Centre $(1, 2)$, radius 3 (1).

Answer: the curve is the circle $(x-1)^2 + (y-2)^2 = 9$, with centre $(1, 2)$ and radius 3.

Answer to Question 3 (4 marks)

Method: to find a Cartesian equation, eliminate the parameter. If one equation is linear in t, make t the subject and substitute. If the equations involve $\sin t$ and $\cos t$, isolate them and use $\sin^2 t + \cos^2 t = 1$ or a double angle formula. Mark scheme as shown in brackets.

a. Use the double angle formula that contains only $\sin t$, then replace $\sin t$ by x. $\cos 2t = 1 - 2\sin^2 t$ (1) so $y = 1 - 2x^2$ (1).

b. $x = \sin t$ lies between -1 and 1; then x^2 lies between 0 and 1. $-1 \le x \le 1$ (1) and $-1 \le y \le 1$ (1). So only the arch of the parabola between $x = -1$ and $x = 1$ is drawn.

Answer to Question 4 (4 marks)

Method: to find where a parametric curve meets a line or an axis, substitute the

expressions for x and y into the equation of the line, solve for t, then substitute each value of t back to get the coordinates. Mark scheme as shown in brackets.

$2t = t^2 - 8$ (1), so $t^2 - 2t - 8 = 0$ (1) and $(t-4)(t+2) = 0$ (1). Points $(16, 8)$ and $(4, -4)$ (1).

Check: $8 = 16 - 8$ and $-4 = 4 - 8$. Answer: the points are $(16, 8)$ and $(4, -4)$.

Answer to Question 5 (4 marks)

Method: in a parametric model, t is usually time. Answer each question by deciding first whether you need a value of t (from a condition on x or y) or a position (by substituting t). Mark scheme as shown in brackets.

a. Greatest height: $10 - 9.8t = 0$ gives $t = 1.02$ s and $y = 2 + \dfrac{100}{19.6} = 7.10$ m (1).
   Landing: $4.9t^2 - 10t - 2 = 0$ gives $t = 2.224$ s (1), so $x = 26.7$ m (1).
b. No air resistance, ball treated as a particle (1).

Answer: the ball rises to about 7.10 m and lands about 26.7 m away.