Worksheet - Parametric equations

Year: Year 13 · 21

Show your working.

Question 1

Question 1 – A table of values *(Section A · Skills practice)*

A curve is defined by $x = 2t + 1$, $y = t^2 - 3$.

a. Complete a table of values for $t = -2, -1, 0, 1, 2$ and plot the points. [3 marks]
b. Find the value of t for which the point is $(7, 6)$. [2 marks]
c. Explain why the point $(0, -3)$ is not on the curve. [2 marks]

Question 2

Question 2 – Eliminating the parameter by substitution *(Section A · Skills practice)*

Find the Cartesian equation of each curve.

a. $x = 3t - 2$, $y = t^2 + 1$ [3 marks]
b. $x = 2t$, $y = \dfrac{8}{t}$ [2 marks]

Question 3

Question 3 – Trigonometric elimination *(Section A · Skills practice)*

Find the Cartesian equation of each curve and describe its shape.

a. $x = 4\cos t$, $y = 3\sin t$ [3 marks]
b. $x = 2 + 3\cos t$, $y = -1 + 3\sin t$ [3 marks]

Question 4

Question 4 – Using a double angle formula *(Section A · Skills practice)*

A curve has parametric equations $x = \sin t$, $y = \cos 2t$ for $0 \le t \le 2\pi$.

a. Show that the Cartesian equation is $y = 1 - 2x^2$. [3 marks]
b. State the range of values of x and of y on the curve. [2 marks]

Question 5

Question 5 – Meeting a line *(Section A · Skills practice)*

The curve C has parametric equations $x = t^2 + 1$, $y = 2t$. It meets the line $y = x - 4$ at two points.

a. Show that the values of t at the intersections satisfy $t^2 - 2t - 3 = 0$. [2 marks]
b. Solve this equation and find the coordinates of the two points. [3 marks]

Question 6

Question 6 – Crossing the axes *(Section A · Skills practice)*

A curve has parametric equations $x = t^2 - 4$, $y = t^3 - 4t$.

a. Find the coordinates of the points where the curve meets the x-axis. [3 marks]
b. Find the coordinates of the points where the curve meets the y-axis. [2 marks]
c. What does your answer tell you about the shape of the curve? [1 mark]

Question 7

Question 7 – A free kick *(Section B · Real-life practice)*

A footballer takes a free kick. The path of the ball is modelled by $x = 15t$, $y = 1 + 12t - 4.9t^2$, where x is the horizontal distance in metres, y the height in metres and t the time in seconds.

a. Find the greatest height reached by the ball. [3 marks]
b. Find the horizontal distance from the kick when the ball lands. [3 marks]
c. A wall of defenders, 2.2 m tall, stands 9 m from the kick. Does the ball clear the wall? [3 marks]
d. State two assumptions of the model. [2 marks]

Question 8

Question 8 – The Ferris wheel *(Section B · Real-life practice)*

A Ferris wheel at a fair turns once every 40 seconds. A passenger car has position $x = 10\sin\left(\frac{\pi t}{20}\right)$, $y = 12 - 10\cos\left(\frac{\pi t}{20}\right)$ where x and y are in metres relative to the ground level foot of the wheel and t is in seconds from the lowest point.

a. Show that the car moves on a circle and state its centre and radius. [3 marks]
b. Find the lowest and highest heights of the car. [2 marks]
c. Find the height of the car when $t = 5$. [2 marks]
d. For how many seconds in each turn is the car more than 17 m above the ground? [4 marks]

Question 9

Question 9 – A drone on a training circuit *(Section B · Real-life practice)*

A drone flies round an elliptical circuit given by $x = 40\cos t$, $y = 25\sin t$ (metres from the control tower at the origin), where t is a parameter.

a. Find the Cartesian equation of the circuit. [2 marks]
b. Show that the squared distance from the tower is $625 + 975\cos^2 t$. [3 marks]
c. Hence find the greatest and least distance from the tower. [3 marks]
d. Find the position when $t = \dfrac{\pi}{3}$. [2 marks]

Question 10

Question 10 – Fireworks *(Section B · Real-life practice)*

A firework rocket is launched from the ground and follows the path $x = 5t$, $y = 60t - 4.9t^2$ (metres, seconds) until it explodes at the top.

a. Find the Cartesian equation of the path. [2 marks]
b. Find the maximum height and the time at which it occurs. [3 marks]
c. Find how far from the launch point the rocket lands if it does not explode. [3 marks]
d. The rocket explodes at the highest point. How far is it then, horizontally, from the launch point? [1 mark]

Question 11

Question 11 – A reflector on a bicycle wheel *(Section B · Real-life practice)*

A reflector is attached to the rim of a bicycle wheel of radius 0.35 m, which rolls along a flat road. The position of the reflector is $x = 0.35(t - \sin t)$, $y = 0.35(1 - \cos t)$ in metres, where t is the angle turned through by the wheel in radians.

t	**0**	$\pi/2$	π	$3\pi/2$	2π
x (m)	?	?	?	?	?
y (m)	?	?	?	?	?

a. Complete the table to 2 decimal places. [4 marks]
b. State the greatest height of the reflector and the distance travelled in one complete turn of the wheel. [2 marks]
c. Explain why the reflector is momentarily at rest relative to the road when $t = 0$ and $t = 2\pi$. [1 mark]

Question 12

Question 12 – A ski jump *(Section B · Real-life practice)*

A skier leaves the end of a horizontal ramp 30 m above a flat landing area. The path is $x = 20t$, $y = 30 - 5t^2$.

a. Show that the Cartesian equation is $y = 30 - \frac{x^2}{80}$. [2 marks]
b. Find the horizontal distance of the landing point from the ramp. [3 marks]
c. Find the height of the skier when 30 m has been covered horizontally. [2 marks]
d. The landing area is a 25 degree slope falling away at the end of the ramp instead. Explain whether the skier would land earlier or later. [1 mark]

Question 13

Question 13 – A satellite and a ground line *(Section B · Real-life practice)*

A small satellite follows the elliptical path $x = 4\cos t$, $y = 3\sin t$ (units of 1000 km). A tracking beam is the line $y = x$.

a. Show that the beam meets the path where $\tan t = \frac{4}{3}$. [3 marks]
b. Find the coordinates of both points of intersection. [4 marks]
c. Calculate the distance between the two points. [2 marks]

Question 14

Question 14 – A boat and a lighthouse *(Section B · Real-life practice)*

A boat sails so that its position relative to a lighthouse at the origin is $x = 3t - 6$, $y = 4t - 5$ (km, with t in hours).

a. Show that the square of the distance from the lighthouse is $25t^2 - 76t + 61$. [3 marks]
b. Find the time at which the boat is closest to the lighthouse and the minimum distance. [4 marks]
c. The lighthouse light can be seen up to 2 km away. Can the sailors see it at their closest approach? [1 mark]

Question 15

Question 15 – Domains of a parametric curve *(Section C · Problem solving)*

Find the Cartesian equation of each curve and state the part of that curve which is traced out.

a. $x = e^t$, $y = e^{2t} + 1$ for all real t. [4 marks]
b. $x = \frac{1}{1+t}$, $y = \frac{t}{1+t}$ for $t \neq -1$. [4 marks]
c. $x = t^2$, $y = t^4 - 2$ for $-1 \leq t \leq 2$. [4 marks]

Question 16

Question 16 – The loop *(Section C · Problem solving)*

A curve is given by $x = t^2 - 1$, $y = t^3 - t$.

a. Show that the curve passes through the origin for two different values of t. [2 marks]
b. Find the coordinates where the curve crosses the x-axis. [2 marks]
c. The curve forms a loop for $-1 \le t \le 1$. Show that the area enclosed is $\frac{8}{15}$ square units. [6 marks]
d. Describe the symmetry of the curve. [1 mark]

Question 17

Question 17 – Tangent and a second meeting point *(Section C · Problem solving)*

The curve C has parametric equations $x = t^2$, $y = t^3$.

a. Show that the Cartesian equation is $y^2 = x^3$. [2 marks]
b. Find the gradient of the tangent at the point where $t = 2$. [3 marks]
c. Find the equation of this tangent. [2 marks]
d. Show that the tangent meets the curve again at the point $(1, -1)$. [4 marks]

Question 18

Question 18 – Fitting a model to drone data *(Section C · Problem solving)*

A drone is tracked and its position (in metres) recorded every second.

t (s)	0	1	2	3	4
x	2	5	8	11	14
y	1	4.5	7	8.5	9

a. Show that the data fit the model $x = 2 + 3t$, $y = 1 + 4t - 0.5t^2$. [3 marks]
b. Find the Cartesian equation of the path. [4 marks]
c. Use the model to predict the position at $t = 8$ and comment on whether the model is realistic. [3 marks]

Question 19

Question 19 – A basketball shot *(Section C · Problem solving)*

A player shoots at a hoop 3.05 m above the ground and 4.5 m horizontally from the player. The ball leaves the hand at a height of 2 m. Its path is modelled by $x = 5.625t$, $y = 2 + 5.25t - 4.9t^2$ (metres and seconds).

a. Find the time at which the ball is 4.5 m from the player horizontally. [2 marks]
b. Find the height of the ball at this time. [2 marks]

c. The hoop has a diameter of 0.45 m (take its centre at height 3.05 m). Decide whether the shot goes in. [2 marks]
d. Find the greatest height of the ball. [3 marks]
e. State two limitations of the model. [2 marks]

Question 20

Question 20 – Challenge: a curve from rational parameters *(Section D · Challenge)*

A curve has parametric equations $x = \dfrac{1 - t^2}{1 + t^2}$, $y = \dfrac{2t}{1 + t^2}$ for all real t.

a. Show that $x^2 + y^2 = 1$. [3 marks]
b. Find the points of the circle corresponding to $t = 0, 1, -1$. [2 marks]
c. Explain why the point $(-1, 0)$ is never reached, although it lies on the circle. [3 marks]

Question 21

Question 21 – Challenge: a figure of eight *(Section D · Challenge)*

A curve has parametric equations $x = \sin t$, $y = \sin 2t$ for $0 \le t \le 2\pi$.

a. Show that $y^2 = 4x^2(1 - x^2)$. [3 marks]
b. Find the values of t for which the curve passes through the origin and explain what this means about its shape. [3 marks]
c. Find the greatest value of y and the corresponding x for $0 \le t \le \dfrac{\pi}{2}$. [3 marks]