Test - Numerical methods

Year: Year 13 · Duration: 40 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	5	4	3	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Change of sign

$f(x) = x^3 - x - 4$.

a. Show that $f(x) = 0$ has a root in $[1, 2]$. [2 marks]
b. Show that the root lies in $[1.7, 1.8]$. [2 marks]

Question 2 (5 marks)

Question 2 – Iteration and Newton-Raphson

The equation $x^3 - x - 4 = 0$ has a root α near 1.8.

a. Use $x_{n+1} = \sqrt[3]{x_n + 4}$ with $x_0 = 1.5$ to find x_1, x_2, x_3 to 4 decimal places. [2 marks]
b. Use Newton-Raphson on $f(x) = x^3 - x - 4$ with $x_0 = 1.8$ to find x_1 and x_2 to 5 decimal places. [3 marks]

Question 3 (4 marks)

Question 3 – The trapezium rule

Use the trapezium rule with 4 strips to estimate $\int_0^1 \sqrt{1 + x^2}\, dx$ to 4 decimal places, and say whether your estimate is too large or too small.

Question 4 (3 marks)

Question 4 – A failure of Newton-Raphson

$f(x) = x^3 - 2x + 2$. Newton-Raphson is applied with $x_0 = 0$. Write down x_1, x_2, x_3 and explain what happens.

Question 5 (4 marks)

Question 5 – Why does the iteration converge?

The iteration $x_{n+1} = g(x_n)$ with $g(x) = \sqrt[3]{x+4}$ is used to solve $x^3 - x - 4 = 0$ with root $\alpha = 1.796$.

a. Find $g'(x)$. [2 marks]
b. Evaluate $g'(\alpha)$ and explain why the iteration converges and what pattern the values follow. [2 marks]