Lesson - Moments

Year: Year 13

A force can make an object **turn** as well as move. This chapter defines the moment of a force and uses moments to solve problems about beams, rods and tilting.

Starter activity

Balance a ruler on your finger. Place a coin at one end and a 3-coin pile at the other end. Where do you need to move the pivot? Write down a rule that connects the weights and the distances. Try again with the coin pile at 10 cm and the single coin at 25 cm from the pivot. What do you notice about the product of weight and distance on each side? Explain in a sentence why a long spanner makes it easier to loosen a tight bolt.

The moment of a force

Definition: moment of a force

The **moment** of a force about a point is its turning effect:

$$\text{moment} = \text{force} \times \text{perpendicular distance from the point to the line of action of the force},$$

measured in newton metres (N m). It can be clockwise or anticlockwise: choose one direction as positive and use it for every force in the question.

Worked example 1

Find the moment of (a) a 15 N force 0.8 m from a hinge at right angles to the rod, (b) a 20 N force at 30° to a rod, acting 0.5 m from the pivot.

(a) $15 \times 0.8 = 12$ N m. (b) Use only the perpendicular component: $20 \sin 30^\circ \times 0.5 = 5$ N m.

Equilibrium of a rigid body

Rule: two conditions for equilibrium

A rigid body is in equilibrium when (1) the resultant force is zero, so the forces in two perpendicular directions each balance, and (2) the sum of the moments about any point is zero: the total clockwise moment equals the total anticlockwise moment.

Worked example 2

A child of mass 30 kg sits 2 m from the pivot of a seesaw. Where must a 40 kg child sit to balance?

$30g \times 2 = 40g \times x$, so $x = 1.5$ m. Mass can replace weight when g cancels, but say so clearly.

Models for beams and rods

Definition: rod, uniform, light and non-uniform

A beam or plank is modelled as a **rod**: a rigid straight object with negligible thickness. A **uniform** rod has its weight acting at its midpoint. A **light** rod has negligible mass. A **non-uniform** rod has a centre of mass that is not at the midpoint. A **smooth** support or peg exerts a force perpendicular to the rod.

Method for a beam problem

Method: beam problems

1. Draw a large diagram. Mark all forces: weights (at the centre of mass of each object), reactions or tensions at the supports, and any applied forces.
2. Take moments about a point where an unknown force acts, because its moment is then zero.
3. Write one moment equation and solve it.
4. Resolve vertically to find the other force.
5. Check by taking moments about a different point.

Worked example 3

A uniform beam AB of length 4 m and mass 20 kg rests on supports at A and B. A 10 kg load is placed 1 m from A. Find the reactions.

Moments about A: $R_B \times 4 = 20g \times 2 + 10g \times 1 = 50g$, so $R_B = 12.5g = 122.5$ N.
Resolving vertically: $R_A + R_B = 30g$, so $R_A = 294 - 122.5 = 171.5$ N.

Worked example 4

A uniform rod AB of length 6 m and mass 12 kg rests on supports at C, 1 m from A, and D, 1.5 m from B. Find the reactions.

Moments about C: $R_D \times 3.5 = 12g \times 2$, so $R_D = 67.2$ N and $R_C = 12g - 67.2 = 50.4$ N.

Worked example 5

A uniform rod of length 1.2 m and mass 3 kg hangs on vertical strings at 0.2 m from each end, with a 4 kg mass 0.3 m from A. Find the tensions.

About the first string (at 0.2 m): $T_2 \times 0.8 = 3g \times 0.4 + 4g \times 0.1 = 1.6g$, so $T_2 = 2g = 19.6$ N and $T_1 = 7g - 19.6 = 49$ N.

Tilting and limiting cases

Rule: about to tilt

When a beam is on the point of tilting about one support, the reaction at the *other* support is zero. Take moments about the support it is tilting about.

Worked example 6

A uniform plank of length 6 m and mass 20 kg rests on supports 1 m from each end. A man of mass 60 kg walks towards one end. How far can he go before it tilts?

Tilting about the nearer support starts when $60g \times x = 20g \times 2$, where x is his distance beyond the support, so $x = 0.667$ m. He can go 0.667 m beyond the support, which is 0.333 m from the end.

Worked example 7

A uniform beam of weight 60 N and length 4 m is supported at one end and at a point 1 m from the other end. What load W at that end makes it tilt?

It tilts when $W \times 1 = 60 \times 1$, so $W = 60$ N.

Non-uniform rods and angled forces

Worked example 8

A non-uniform rod of mass 5 kg and length 2 m balances on a pivot 0.9 m from A when a 2 kg mass hangs at A. Find the centre of mass.

Let the centre of mass be d from A. Moments about the pivot:
$2g \times 0.9 = 5g \times (d - 0.9)$, so $d = 1.26$ m.

Worked example 9

A uniform rod of mass 5 kg and length 2 m is hinged at A and held horizontal by a string at B making 40° with the rod. Find the tension. A wheelbarrow carries 40 kg at 0.6 m from the axle; find the lift F at 1.4 m.

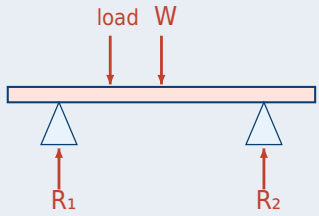
The perpendicular component of the tension is $T\sin 40^\circ$, so $T\sin 40^\circ \times 2 = 5g \times 1$ and $T = 38.1$ N. For the wheelbarrow $1.4F = 40g \times 0.6$, so $F = 168$ N.

Watch out

Use the *perpendicular* distance, not the distance along a slanting line. A uniform rod's weight acts at the midpoint, not at the end. Include the mass of the rod itself. Do not mix clockwise and anticlockwise moments. In the limiting case the reaction at the

support about to lift off is zero.

Note: the lever



Worked example 10: Taking moments about the unknown force

A uniform plank of length 3 m and mass 12 kg lies on two supports at its ends. A 20 kg load is 1 m from one end A. Find the reaction at B.

Take moments about A so that R_A has no moment:
$R_B \times 3 = 12g \times 1.5 + 20g \times 1 = 38g$, giving $R_B = 12.67g = 124.1$ N. Check by moments about B: $R_A \times 3 = 12g \times 1.5 + 20g \times 2 = 58g$, $R_A = 19.33g = 189.5$ N. Then $R_A + R_B = 313.6 = 32g$ as required.

Worked example 11: A moment with a force at an angle

A 50 N force acts on a door handle at an angle of 40° to the door, 0.9 m from the hinge. Find the moment about the hinge.

Only the component perpendicular to the door contributes: $50 \sin 40^\circ = 32.1$ N. Moment $= 32.1 \times 0.9 = 28.9$ N m. Equivalently, the perpendicular distance from the hinge to the line of the force is $0.9 \sin 40^\circ = 0.579$ m, and $50 \times 0.579 = 28.9$ N m. Pushing at right angles ($50 \times 0.9 = 45$ N m) is more effective.

Worked example 12: A beam supported at one point

A uniform beam of mass 10 kg and length 4 m is supported at a point 1 m from end A. A mass m hangs at A. Find m for balance.

The centre of the beam is 1 m on the other side of the support. Moments:
$m \times 1 = 10 \times 1$, so $m = 10$ kg. The support force is $20g = 196$ N. If $m = 15$ kg, the beam would rotate with A going down.

Worked example 13: Two different pivots

A seesaw of length 4 m with masses 30 kg at one end and 50 kg at the other balances at a distance x from the 50 kg end. Find x for a light seesaw, and for a seesaw of mass 20 kg.

Light seesaw: $50x = 30(4 - x)$, so $80x = 120$ and $x = 1.5$ m. With a uniform seesaw of mass 20 kg, the centre is 2 m from each end. Moments about the pivot:
$50x = 30(4 - x) + 20(2 - x)$, giving $50x = 160 - 50x$ and $x = 1.6$ m. The mass of the

seesaw moves the pivot 0.1 m further from the 50 kg child.

Rule: how to take moments

Choose a direction (say anticlockwise) as positive. For each force write force $\times$ perpendicular distance and give it the right sign. Choose the point about which you take moments so that as many unknown forces as possible pass through it. For a beam in equilibrium, the sum of all moments is zero about *every* point, so you can always check using a second point.

Watch out: common errors

- Using the length along the slanting force instead of the perpendicular distance.
- Forgetting the weight of the rod itself, or placing it at the end instead of the centre.
- Assuming both supports carry equal loads when there is a load on the beam.
- In a tilting problem, taking moments about the wrong support: use the support about which the beam is tilting.

Did you know?

Archimedes of Syracuse (about 287-212 BC) is reported to have said "Give me a place to stand and I will move the Earth", describing the power of the lever. His law of the lever, that two weights balance when their distances from the pivot are in inverse proportion to the weights, is the first example of the principle of moments that engineers still use to design cranes, bridges and playground equipment.

Otto's tip

Take moments about the unknown force you do not want, then resolve vertically for the other one.