Answers - Moments

Year: Year 13

Answer to Question 1

Method: the moment of a force about a point is force $\times$ perpendicular distance from the point to the line of action. For a body in equilibrium, the resultant force is zero and the total clockwise moment equals the total anticlockwise moment about any point.

a. The force is perpendicular to the spanner, so the distance is simply 0.35 m. Moment $= 24 \times 0.35 = 8.4$ N m. So the moment is 8.4 N m.
b. Only the component of the force perpendicular to the rod turns it, so use $40 \sin 50^\circ$. The perpendicular component of the force is $40 \sin 50^\circ = 30.64$ N, so the moment is $30.64 \times 0.6 = 18.4$ N m. Equivalently, the perpendicular distance to the line of action is $0.6 \sin 50^\circ$.
c. Rearrange moment $=$ force $\times$ distance. Distance $= \dfrac{30}{75} = 0.4$ m. So the line of action is 0.4 m from O.

Answer to Question 2

a. Moments about the pivot: $28g \times 1.8 = 36g \times x$, so $x = \dfrac{50.4}{36} = 1.4$ m.
b. The 36 kg and 18 kg children are at 1.4 m and 1.9 m. Moments: $28g \times d = 36g \times 1.4 + 18g \times 1.9 = 50.4g + 34.2g = 84.6g$, so $d = \dfrac{84.6}{28} = 3.02$ m. (This may be more than half the length of a real seesaw!)

Answer to Question 3

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. Forces: R_A and R_B upwards at the ends; $30g$ down at the midpoint (2.5 m from A); $15g$ down at 2 m from A.
b. Moments about A: $R_B \times 5 = 30g \times 2.5 + 15g \times 2 = 105g$. So $R_B = 21g = 205.8$ N.
c. Vertically: $R_A + R_B = 45g = 441$, so $R_A = 441 - 205.8 = 235.2$ N.

Answer to Question 4

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. The two supports and the two end lengths make up the whole rod. $CD = 8 - 1 - 2 = 5$ m.
b. Take moments about C. The weight acts 3 m from C (the midpoint is 4 m from A). $R_D \times 5 = 16g \times 3$, so $R_D = 9.6g = 94.1$ N. Vertically $R_C = 16g - 94.08 = 62.7$ N. Check with moments about D: $R_C \times 5 = 16g \times 2$ gives $R_C = 6.4g = 62.7$ N.

Answer to Question 5

a. When the plank is about to tilt about one support, it is just losing contact with the other support, so the reaction there is zero.
b. Take moments about the nearer support. The weight of the plank acts 2 m from the support: $50g \times x = 20g \times 2$, so $x = 0.8$ m.
c. The overhang is only 0.5 m, which is less than 0.8 m. So he can reach the end without the plank tilting.

Answer to Question 6

Let T_1 be at 0.25 m from A and T_2 at 1.25 m from A. The centre of the rod is 0.5 m to the right of the first string, and the particle is 0.15 m to the left of it, so its moment opposes the moment of the rod. Moments about the first string: $T_2 \times 1.0 = 4g \times 0.5 - 6g \times 0.15 = 2g - 0.9g = 1.1g$, so $T_2 = 10.78$ N. Vertically: $T_1 + T_2 = 10g = 98$, so $T_1 = 87.2$ N.

Answer to Question 7

a. Moments about the axle: $F \times 1.3 = 45g \times 0.5$, so $F = \dfrac{220.5}{1.3} = 169.6$ N.
b. Vertically: $R + F = 45g = 441$, so $R = 441 - 169.6 = 271.4$ N.
c. The load is now 0.3 m from the axle: $F = \dfrac{441 \times 0.3}{1.3} = 101.8$ N. Moving the load towards the wheel reduces the force needed from the gardener, which is why heavy loads are placed over the axle.

Answer to Question 8

a. The board would rotate about B with C going down and A rising. To prevent A rising, the bolt must pull down on it.
b. Moments about B: the diver is 2.2 m from B; the centre of the board is 1.5 m from A, i.e. 0.7 m from B on the same side as C. So $F_A \times 0.8 = 60g \times 2.2 + 24g \times 0.7 = 132g + 16.8g = 148.8g$, $F_A = 186g = 1822.8$ N.
c. Vertically: $R_B = F_A + 60g + 24g = 186g + 84g = 270g = 2646$ N.

Answer to Question 9

a. Supports at $x = 1$ (C) and $x = 5$ (D). Moments about D:
$R_C \times 4 = 40g \times 2 + 45g \times 3 = 215g$, so $R_C = 53.75g = 526.75$ N.
$R_D = 85g - 526.75 = 306.25$ N.
b. The most dangerous position is at an end. At A, the girl is 1 m beyond C and the plank centre is 2 m from C. For tilting about C we need $45g \times 1 \geq 40g \times 2$, i.e. $45 \geq 80$. This is false. By symmetry the same is true at B. So the plank never tilts.

Answer to Question 10

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force. A hinge can exert a force in any direction, so give it a horizontal and a vertical component.

a. Moments about A: $T \sin 50^\circ \times 2.4 = 8g \times 1.2$. So $T = \dfrac{94.08}{2.4 \sin 50^\circ} = \dfrac{94.08}{1.8385} = 51.2$ N.
b. Horizontal: the cable pulls $T \cos 50^\circ = 32.9$ N towards the wall; the hinge force must balance this, so $X = 32.9$ N away from the wall. Vertical: $Y + T \sin 50^\circ = 8g$, so $Y = 78.4 - 39.2 = 39.2$ N upwards.

Answer: $T \approx 51.2$ N; the hinge pushes with about 32.9 N horizontally and 39.2 N vertically upwards.

Answer to Question 11

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. The centre of mass must be on the other side of the pivot from the child. Let the centre of mass be d m from A. Moments about the pivot: $12g \times 1.8 = 20g(d - 1.8)$. So $d - 1.8 = 1.08$ and $d = 2.88$ m. So the centre of mass is 2.88 m from A, not at the midpoint: the plank is non-uniform.
b. The centre of mass (2.88 m from A) and the child at B (5 m from A) are both on the same side of the old pivot, so the plank would tip: the pivot must move towards B, between the centre of mass and B. Let the pivot be p m from A. Moments about the pivot: $20g(p - 2.88) = 8g(5 - p)$, so $20p - 57.6 = 40 - 8p$, $28p = 97.6$ and $p = 3.49$ m. So the pivot must be placed about 3.49 m from A. Check: $20 \times 0.606 = 12.1$ and $8 \times 1.514 = 12.1$.

Answer to Question 12

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take

moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. The weight of the uniform rule acts at the 50 cm mark, 10 cm from the pivot; the mass is 30 cm from the pivot. Centre of the rule at 50 cm. Moments about the pivot (40 cm): $m \times 30 = 120 \times 10$, so $m = 40$ g. So the small mass is 40 g.
b. The 40 g mass at 0 cm is x cm from the pivot and the rule's weight is $(50 - x)$ cm from it. Let the pivot be at x cm. Moments: $40x = 120(50 - x)$, so $160x = 6000$ and $x = 37.5$ cm. So the pivot must be moved to the 37.5 cm mark.

Answer to Question 13

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. Taking moments about A removes the reaction and the friction at A. Take moments about A: $(T\cos 20^\circ) \times 8 = 600g \times 4$, so $T\cos 20^\circ = 300g$, half the weight. So the cable carries half the weight vertically.
b. Divide the vertical component by $\cos 20^\circ$. $T = \dfrac{300 \times 9.8}{\cos 20^\circ} = \dfrac{2940}{0.9397} = 3128.7$ N. So the tension is about 3130 N.
c. Resolve horizontally: the only other horizontal force is at A. Horizontal component $= T\sin 20^\circ = 1070$ N (since $2940\tan 20^\circ = 1070$). Friction at A acts horizontally to balance it. Without friction the end A would slide along the ground.

Answer to Question 14

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. Taking moments about A removes the unknown force of the hand at A. The rod's weight acts at its midpoint, 1.5 m from A. Moments about the hand at A: $F_2 \times 0.3 = 0.8g \times 1.5 + 10 \times 3 = 7.84 \times 1.5 + 30$. Since $0.8g = 7.84$ N, $11.76 + 30 = 41.76$. So $F_2 = 139.2$ N upwards. So the hand 0.3 m from A pushes up with about 139 N.
b. Resolve vertically, taking upwards as positive. Vertically (taking up as positive): $F_2 + F_1 = 7.84 + 10$, so $F_1 = 17.84 - 139.2 = -121.4$. The hand at A pushes down with 121 N. The negative sign means this hand pushes downwards, acting like a lever.

Answer to Question 15

a. Moments about D (everything is to the left of D except nothing): distances from D: plank 1.8, worker 3.2, tool box 0.7, C at 3.6.

$R_C \times 3.6 = 30g \times 1.8 + 70g \times 3.2 + 20g \times 0.7 = 292g$, so $R_C = 81.1g = 794.9$ N.
$R_D = 120g - 794.9 = 381.1$ N.

b. Worker at B is 0.6 m to the right of D.
$R_C \times 3.6 = 30g \times 1.8 + 20g \times 0.7 - 70g \times 0.6 = 26g$, so $R_C = 7.22g = 70.8$ N and $R_D = 1176 - 70.8 = 1105.2$ N. Since $R_C > 0$, the plank does not tilt.

c. The plank is uniform (so its weight acts at the midpoint) and the worker and tool box can be treated as particles; also the supports exert only vertical forces.

Answer to Question 16

a. Moments about A: $T\sin 40^\circ \times 2 = 10g \times 1 + 8g \times 1.5 = 98 + 117.6 = 215.6$. So $T = \dfrac{215.6}{2\sin 40^\circ} = \dfrac{215.6}{1.2856} = 167.7$ N.

b. Horizontal: $X = T\cos 40^\circ = 128.5$ N (away from the wall). Vertical: $Y = 18g - T\sin 40^\circ = 176.4 - 107.8 = 68.6$ N upwards.

c. $|H| = \sqrt{128.5^2 + 68.6^2} = 145.6$ N at $\tan^{-1}\dfrac{68.6}{128.5} = 28.1^\circ$ above the horizontal.

d. $T = \dfrac{215.6}{2\sin 25^\circ} = 255.1$ N. A shallow cable angle gives a smaller perpendicular component, so the tension needed is much bigger.

Answer to Question 17

a. Total moment about A of the weights $= g(6 \times 1 + 2 \times 0.2 + 3 \times 0.6 + 5 \times 1.1 + 4 \times 1.8) = g(6 + 0.4 + 1.8 + 5.5 + 7.2) = 20.9g$. Total mass $= 20$ kg. Balance point $\bar{x} = \dfrac{20.9}{20} = 1.045$ m.

b. Reaction $= 20g = 196$ N.

c. Without the 4 kg mass: moment $= 13.7g$, mass 16 kg, so $\bar{x} = \dfrac{13.7}{16} = 0.856$ m.
The pivot must move $1.045 - 0.856 = 0.189$ m towards A.

Answer to Question 18

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. Weight $= 180 + 220 = 400$ N. Moments about A: $400 \times d = 220 \times 4$, so $d = 2.2$ m.

b. Person weight $= 70g = 686$ N. Moments about A:
$R_B \times 4 = 400 \times 2.2 + 686 \times 1 = 1566$, so $R_B = 391.5$ N.
$R_A = 1086 - 391.5 = 694.5$ N.

c. $R_B \le 800$: $4 \times 800 = 3200 \ge 880 + 686x$, so $x \le 3.39$ m. (And check R_A stays positive.)

Answers: the plank weighs 400 N with centre of mass 2.2 m from A; the scales then read 694.5 N and 391.5 N; the person must stay within about 3.39 m of A.

Answer to Question 19

a. Sliding: $P = \mu R = 0.4 \times 50g = 196$ N.
b. Tipping about the front lower edge: the weight acts 0.3 m horizontally from the edge. The tip occurs when $P \times 1.0 = 50g \times 0.3$, so $P = 147$ N.
c. Tipping needs only 147 N, which is less than 196 N, so the crate tips first.
d. Let the height be h. Sliding when $P \geq 196$. It tips if $Ph \geq 147$, i.e. $P \geq 147/h$. For sliding first we need $147/h \geq 196$, so $h \leq 0.75$ m. Pushing at or below 0.75 m makes the crate slide.

Answer to Question 20

a. The centre of the plank is $L - d$ from the nearer support. Moments about that support when the plank is about to tilt: $mgx = Mg(L - d)$, so $x = \dfrac{M(L-d)}{m}$.
b. The person can walk beyond the support by at most d. The plank does not tilt if the distance needed for tilting is larger than d: $\dfrac{M(L-d)}{m} > d \iff ML - Md > md \iff d$. So the person reaches the end without tilting exactly when d.
c. $\dfrac{ML}{M+m} = \dfrac{40 \times 3}{85} = 1.41$ and $d = 1$, so there is no tilting, as found in the walking across a plank question. (With $M = 20$, $m = 60$, $L = 3$, the limit is 0.75 m, so a support 1 m from the end would tilt.)

Answer to Question 21

Method: draw a force diagram (the weight of a uniform body acts at its midpoint). Take moments about a point where an unknown force acts, so that this force disappears from the equation, then resolve vertically to find the remaining force.

a. The vertical component of the tension is $T \sin\theta$, acting at distance $2a$ from A; the weight acts at distance a. Moments about A: $T \sin\theta \times 2a = W \times a$, so $T = \dfrac{W}{2\sin\theta}$. Notice that a cancels, so the length does not matter.
b. The tension is smallest when the denominator $2\sin\theta$ is largest. The minimum occurs when $\sin\theta = 1$, i.e. $\theta = 90^\circ$, with $T = \dfrac{W}{2}$. So a vertical string gives the smallest tension, half the weight.
c. Substitute $W = 100$ and require $T \leq 80$. $\dfrac{100}{2\sin\theta} \leq 80$ gives $\sin\theta \geq 0.625$, so

$\theta \geq 38.7^\circ$. So the string must make at least 38.7° with the rod.