Test answers - Forces in two dimensions and friction

Year: Year 13 · Duration: 40 minutes · /20

Answer to Question 1 (4 marks)

Method: resolve both forces east and north, add the components, then use Pythagoras and the inverse tangent. Mark scheme: 1 mark for each component, 1 for the magnitude, 1 for the direction.

East component $= 15 + 10\cos 50^\circ = 21.43$ N (1); north component $= 10\sin 50^\circ = 7.66$ N (1). Magnitude $= \sqrt{21.43^2 + 7.66^2} = 22.8$ N (1). Direction $\tan^{-1}\dfrac{7.66}{21.43} = 19.7^\circ$ north of east (1).

Answer: the resultant is about 22.8 N on a bearing 19.7° north of east.

Answer to Question 2 (4 marks)

Method: the picture is in equilibrium, so the horizontal components of the tensions balance each other and the vertical components together balance the weight. Mark scheme: 1 mark for each equation, 1 mark for each tension.

Horizontal: $T_1\cos 35^\circ = T_2\cos 55^\circ$ (1). Vertical: $T_1\sin 35^\circ + T_2\sin 55^\circ = 60$ (1). Since $\cos 55^\circ = \sin 35^\circ$, the first equation gives $T_1 = T_2\tan 35^\circ$. Substituting, $T_2(\tan 35^\circ\sin 35^\circ + \sin 55^\circ) = 60$, so $1.2208T_2 = 60$ and $T_2 = 49.1$ N (1), $T_1 = 34.4$ N (1). Check: $34.41\sin 35^\circ + 49.15\sin 55^\circ = 19.74 + 40.26 = 60$.

Answer: the steeper string carries more of the weight: $T_2 \approx 49.1$ N and $T_1 \approx 34.4$ N.

Answer to Question 3 (5 marks)

Method: friction can take any value up to the limiting value μR. The object moves only if the applied force is greater than μR; once it moves, friction equals μR and acts against the motion. Mark scheme as shown in brackets.

a. The vertical component of the tension, $40\sin 30^\circ = 20$ N, lifts the crate a little, so R is less than the weight. $R = 12\times 9.8 - 40\sin 30^\circ = 117.6 - 20 = 97.6$ N (2). So $R = 97.6$ N.

b. Compare the horizontal pull with the limiting friction. $F_{max} = 0.3\times 97.6 = 29.3$ N (1). Horizontal pull $40\cos 30^\circ = 34.6$ N > 29.3 N, so the crate moves (1). So the crate moves and friction is 29.3 N.

c. Apply $F = ma$ horizontally. $a = \dfrac{34.64 - 29.28}{12} = 0.447$ m s^{-2} (1). So the crate accelerates at about 0.447 m s^{-2}.

Answer to Question 4 (4 marks)

Method: on a slope, resolve parallel and perpendicular to the plane. Perpendicular: $R = mg\cos\alpha$ (plus any other perpendicular components). Parallel: use $F = ma$, with friction $F = \mu R$ acting against the motion or the likely motion. Mark scheme: 2 marks for each part.

a. The block stays at rest only if $\tan\alpha \leq \mu$, because then $mg\sin\alpha \leq \mu mg\cos\alpha$. $\tan 35^\circ = 0.700 > 0.4$, so friction cannot hold it (2). So the block slides down the plane.
b. Friction is limiting and acts up the slope; $F = ma$ along the slope gives $mg\sin 35^\circ - 0.4mg\cos 35^\circ = ma$ and the mass cancels. $a = g(\sin 35^\circ - 0.4\cos 35^\circ) = 9.8(0.5736 - 0.3277) = 2.41$ m s^{-2} (2). So the block accelerates at about 2.41 m s^{-2}.

Answer to Question 5 (3 marks)

Method: write $F = ma$ for each particle in its own direction of motion; the string is light and inextensible, so the tension and the acceleration are the same for both. Add the equations to eliminate T. Mark scheme: 1 mark for the equations, 1 for a, 1 for T.

Hanging: $2g - T = 2a$. Table: $T = 3a$ (1). So $2g = 5a$ and $a = 3.92$ m s^{-2} (1). $T = 3 \times 3.92 = 11.8$ N (1).

Answer: the system accelerates at 3.92 m s^{-2} and the tension is about 11.8 N.