Worksheet - Forces in two dimensions and friction

Year: Year 13 · 21

Show your working.

Question 1

Question 1 – Resolving a force *(Section A · Skills practice)*

A force of 30 N acts at 40° above the horizontal.

a. Find its horizontal component. [2 marks]
b. Find its vertical component. [2 marks]
c. Write the same force as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$ with $\mathbf{i}$ horizontal and $\mathbf{j}$ vertically upwards. [1 mark]

Question 2

Question 2 – The resultant of two forces *(Section A · Skills practice)*

Two forces act on a particle at the origin. One is 12 N along the positive x-axis. The other is 9 N at 60° above the positive x-axis.

a. Find the total horizontal component and the total vertical component. [3 marks]
b. Find the magnitude of the resultant. [2 marks]
c. Find the angle the resultant makes with the x-axis. [2 marks]

Question 3

Question 3 – Forces as vectors *(Section A · Skills practice)*

Two forces $\mathbf{F}_1 = (4\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_2 = (-\mathbf{i} + 5\mathbf{j})$ N act on a particle of mass 2 kg.

a. Find the resultant force. [1 mark]
b. Find its magnitude and the angle it makes with $\mathbf{i}$. [3 marks]
c. Find the magnitude of the acceleration of the particle. [2 marks]

Question 4

Question 4 – Two strings *(Section A · Skills practice)*

An 80 N weight hangs in equilibrium from two light strings. One string makes an angle of 40° with the horizontal and the other makes 50° with the horizontal, on opposite sides of the weight.

a. Draw a force diagram. [1 mark]

b. By resolving horizontally and vertically, form two equations for the tensions T_1 and T_2. [3 marks]
c. Solve them to find T_1 and T_2. [3 marks]

Question 5

Question 5 – Friction on a horizontal floor *(Section A · Skills practice)*

A box of mass 15 kg rests on a rough horizontal floor. The coefficient of friction is 0.4. Take $g = 9.8$ m s^{-2}.

a. Find the normal reaction. [1 mark]
b. Find the least horizontal force that makes the box move. [2 marks]
c. A horizontal force of 100 N is applied. Find the acceleration. [3 marks]

Question 6

Question 6 – A rough and a smooth slope *(Section A · Skills practice)*

A block of mass 6 kg slides down a plane inclined at 25° to the horizontal. Take $g = 9.8$ m s^{-2}.

a. The plane is smooth. Find the acceleration. [2 marks]
b. The plane is rough with coefficient of friction 0.3. Find the normal reaction and the acceleration. [4 marks]

Question 7

Question 7 – Pulling a sledge *(Section B · Real-life practice)*

A child pulls a sledge of mass 25 kg across level snow using a rope inclined at 30° above the horizontal. The coefficient of friction between the sledge and the snow is 0.15. The sledge moves at constant speed. Take $g = 9.8$ m s^{-2}.

a. Draw a force diagram. [2 marks]
b. Show that the normal reaction is $R = 245 - 0.5P$, where P is the tension in the rope. [2 marks]
c. Find P. [4 marks]
d. The child now holds the rope at 60° above the horizontal. Explain without calculation whether P increases or decreases. [1 mark]

Question 8

Question 8 – A hanging sign *(Section B · Real-life practice)*

A shop sign of weight 60 N hangs from two equal light cables, each making an angle of 35° with the horizontal.

a. Show that the tension in each cable is $T = \dfrac{30}{\sin 35^\circ}$. [3 marks]
b. Calculate T. [1 mark]
c. The shop owner wants the tension to be at most 40 N. Find the least angle the cables must make with the horizontal. [3 marks]

Question 9

Question 9 – Parking on a hill *(Section B · Real-life practice)*

A car of mass 900 kg is parked on a road inclined at 12° to the horizontal with its handbrake on. Take $g = 9.8$ m s^{-2}.

a. Find the friction force needed to hold the car in equilibrium. [2 marks]
b. Find the normal reaction. [2 marks]
c. Find the least coefficient of friction which allows the car to stay at rest. [2 marks]
d. On an icy day $\mu = 0.1$. Find the acceleration of the car if it begins to slide. [3 marks]

Question 10

Question 10 – Pushing a lawnmower *(Section B · Real-life practice)*

A gardener pushes a lawnmower of mass 20 kg along a lawn at constant speed. The handle is inclined at 35° below the horizontal, so the push P acts down the handle. The coefficient of friction is 0.3. Take $g = 9.8$ m s^{-2}.

a. Draw a force diagram. [1 mark]
b. Show that $R = 196 + P\sin 35^\circ$. [2 marks]
c. Find P. [4 marks]
d. State one way in which the gardener could reduce the push required. [1 mark]

Question 11

Question 11 – A skier on a piste *(Section B · Real-life practice)*

A skier of mass 70 kg starts from rest and skis down a slope inclined at 20° to the horizontal. The coefficient of friction between skis and snow is 0.1. Take $g = 9.8$ m s^{-2} and ignore air resistance.

a. Find the acceleration of the skier. [4 marks]
b. Find the speed after 5 seconds. [1 mark]
c. Find the distance travelled in the first 5 seconds. [2 marks]
d. State one reason why the skier might not reach the speed calculated if air resistance were included. [1 mark]

Question 12

Question 12 – Crates on a table and a hanging load *(Section B · Real-life practice)*

A crate of mass 5 kg rests on a rough horizontal table with coefficient of friction 0.25. A light inextensible string connects it over a smooth pulley at the edge of the table to a hanging crate of mass 3 kg. The system is released from rest. Take $g = 9.8$ m s^{-2}.

a. Write down an equation of motion for each crate. [3 marks]
b. Find the acceleration. [3 marks]
c. Find the tension in the string. [2 marks]
d. State what the phrase "light inextensible string" allows you to assume. [1 mark]

Question 13

Question 13 – Two tugs and a barge *(Section B · Real-life practice)*

Two tugs pull a barge using ropes. Each rope exerts a force of 4000 N and makes an angle of 20° with the line along the centre of the river, one on each side. The barge moves along the centre line.

a. Show that the forces perpendicular to the centre line balance. [2 marks]
b. Find the resultant forward force. [2 marks]
c. The water exerts a resistance of 6200 N. Find the acceleration of the barge, which has mass 90 000 kg. [3 marks]

Question 14

Question 14 – Towing a caravan *(Section B · Real-life practice)*

A car of mass 1200 kg tows a caravan of mass 800 kg along a straight horizontal road. The engine produces a driving force of 3000 N. The resistances to motion are 400 N on the car and 300 N on the caravan.

	Mass (kg)	Resistance (N)
Car	1200	400
Caravan	800	300

a. Find the acceleration of the car and caravan. [3 marks]
b. Find the tension in the tow bar. [3 marks]
c. The driver reaches a steady speed. Find the driving force needed and the tension then. [2 marks]

Question 15

Question 15 – A force on a slope *(Section C · Problem solving)*

A block of mass 10 kg is on a rough plane inclined at 30° to the horizontal with coefficient of friction 0.3. A horizontal force P newtons acts on the block in the vertical

plane containing a line of greatest slope, pushing the block towards the plane. Take $g = 9.8$ m s^{-2}.

a. Show that the normal reaction is $R = 98\cos 30^\circ + P\sin 30^\circ$. [3 marks]
b. The block is on the point of sliding **up** the plane. Find P. [5 marks]
c. The block is on the point of sliding **down** the plane. Find P. [4 marks]
d. State the range of values of P for which the block stays at rest. [1 mark]

Question 16

Question 16 – A hanging basket *(Section C · Problem solving)*

A garden centre displays a hanging basket of weight 40 N. A light string attached to the basket makes an angle of 25° with the vertical, and a horizontal force H holds the basket in equilibrium. The basket is treated as a particle.

a. Show that the tension is $T = \dfrac{40}{\cos 25^\circ}$. [3 marks]
b. Find T and H. [3 marks]
c. The angle is increased to 40° with the same weight. Find the new H. [2 marks]
d. The string can support at most 100 N. Find the largest angle to the vertical for which the basket can be held. [3 marks]

Question 17

Question 17 – Block and counterweight on a slope *(Section C · Problem solving)*

A block of mass 4 kg lies on a rough plane inclined at 30° to the horizontal, with coefficient of friction 0.2. It is connected by a light inextensible string passing over a smooth pulley at the top of the plane to a hanging block of mass 3 kg. The system is released from rest. Take $g = 9.8$ m s^{-2}.

a. Show that the 4 kg block moves up the slope. [3 marks]
b. Find the acceleration of the blocks. [5 marks]
c. Find the tension in the string. [2 marks]
d. After 2 seconds the string breaks. Describe the subsequent motion of the 4 kg block. [2 marks]

Question 18

Question 18 – Finding unknown forces *(Section C · Problem solving)*

Three forces $\mathbf{F}_1 = (p\mathbf{i} + 3\mathbf{j})$ N, $\mathbf{F}_2 = (2\mathbf{i} + q\mathbf{j})$ N and $\mathbf{F}_3 = (-4\mathbf{i} - 6\mathbf{j})$ N act on a particle and keep it in equilibrium.

a. Find p and q. [3 marks]
b. $\mathbf{F}_3$ is removed. The particle has mass 0.5 kg. Find its acceleration as a vector. [2 marks]
c. Find the magnitude and direction of the acceleration. [3 marks]

d. Find the speed of the particle 3 seconds after $\mathbf{F}_3$ is removed, if it started from rest. [2 marks]

Question 19

Question 19 – Estimating a coefficient of friction *(Section C · Problem solving)*

A student places a wooden block on a board and slowly raises one end until the block starts to slide. The angle θ at which sliding begins is recorded in five trials.

Trial	1	2	3	4	5
Angle θ	17°	19°	18°	20°	18°

a. Show that at the point of sliding $\mu = \tan\theta$. [3 marks]
b. Find the mean angle and estimate μ. [3 marks]
c. The student then uses a 2.5 kg block on a slope of 15°. Using your estimate, state whether it slides. [2 marks]
d. State two assumptions of the model and suggest one improvement to the experiment. [2 marks]

Question 20

Question 20 – Challenge: the easiest way to drag a crate *(Section D · Challenge)*

A crate of mass m is dragged along a rough horizontal floor with coefficient of friction μ at constant speed by a rope at angle θ above the horizontal.

a. Show that the tension is $P = \dfrac{\mu m g}{\cos\theta + \mu \sin\theta}$. [4 marks]
b. Show that P is smallest when $\tan\theta = \mu$. [4 marks]
c. For $m = 30$ kg and $\mu = 0.4$ find the best angle and the least force. [3 marks]

Question 21

Question 21 – Challenge: the angle of friction *(Section D · Challenge)*

A block of mass m rests on a rough plane inclined at angle α to the horizontal; the coefficient of friction is μ.

a. Prove that the block remains at rest if and only if $\tan\alpha \le \mu$. [4 marks]
b. Find the largest angle for which a block stays at rest when $\mu = 0.45$. [2 marks]
c. When $\alpha = 30^\circ$ and $\mu = 0.45$ the block slides. Find its acceleration. [3 marks]