Test - Differentiation: chain, product and quotient rules

Year: Year 13 · Duration: 40 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Chain and product rules

Differentiate with respect to x.

a. $y = \ln(4x^2 + 1)$ [2 marks]
b. $y = x^3 e^{-2x}$, giving your answer in the form $x^2 e^{-2x}(a + bx)$ [2 marks]

Question 2 (4 marks)

Question 2 – A quotient and its stationary points

$y = \dfrac{x^2}{2x + 1}$ for $x \neq -\frac{1}{2}$.

a. Show that $\dfrac{dy}{dx} = \dfrac{2x(x + 1)}{(2x + 1)^2}$. [2 marks]
b. Find the coordinates of the stationary points. [2 marks]

Question 3 (4 marks)

Question 3 – An implicit curve

A curve has equation $x^2 + 2xy + 3y^2 = 18$.

a. Show that the point $(3, 1)$ lies on the curve and find $\dfrac{dy}{dx}$ there. [3 marks]
b. Find the equation of the tangent at $(3, 1)$. [1 mark]

Question 4 (4 marks)

Question 4 – A parametric curve

A curve is given by $x = t^2 + 2$, $y = 2t^3 - t$.

a. Find $\frac{dy}{dx}$ in terms of t. [2 marks]

b. Find the equation of the normal at the point where $t = 1$. [2 marks]

Question 5 (4 marks)

Question 5 – Rates of change

The volume of a spherical balloon increases at 100 $\text{cm}^3\ \text{s}^{-1}$. Find the rate of increase of the radius and of the surface area when $r = 5$ cm.