Test answers - Conditional probability and the Normal distribution

Year: Year 13 · Duration: 40 minutes · /20

Answer to Question 1 (4 marks)

Mark scheme: 1 mark for each of parts a) and b), 2 marks for part c) (1 for a correct comparison, 1 for the conclusion).

a. Use the gym column total out of 80. $\frac{42}{80} = 0.525$. So $P(\text{gym}) = 0.525$.

b. Given gym, divide by the 42 gym users. $\frac{24}{42} = \frac{4}{7} = 0.571$. So $P(\text{Year 13} \mid \text{gym}) \approx 0.571$.

c. Compare the joint probability with the product of the two probabilities. $P(\text{gym})P(\text{Y12}) = 0.525 \times 0.5 = 0.2625$ but $P(\text{gym} \cap \text{Y12}) = \frac{18}{80} = 0.225$. These differ, so the events are **not independent**. (Alternatively $P(\text{gym} \mid \text{Y12}) = 0.45 \neq 0.525$.) Year 12 students are less likely to go to the gym.

Answer to Question 2 (5 marks)

Mark scheme: a) 1 mark for the method (two paths added), 1 mark for 0.08; b) 1 mark for the conditional probability method, 1 mark for 0.75; c) 1 mark.

a. Add the two paths bus then late and bicycle then late. $P(L) = 0.6 \times 0.1 + 0.4 \times 0.05 = 0.06 + 0.02 = 0.08$. So Tom is late on 8% of days.

b. Divide the path bus then late by the total probability of being late. $P(\text{bus} \mid L) = \frac{0.06}{0.08} = 0.75$. So when Tom is late, he took the bus three times out of four.

c. He is not late with probability $1 - 0.08 = 0.92$ each day, and the days are independent. $0.92^5 = 0.659$. So he is on time all week with probability about 0.659.

Answer to Question 3 (4 marks)

Mark scheme: a) 1 mark, b) 1 mark, c) 1 mark for $z = -0.8416$ and 1 mark for k.

a. Standardise: $z = \frac{70 - 64}{9} = 0.667$, then take the upper tail. $P(X > 70) = P(Z > 0.667) = 0.2525$. So about 25% of tomatoes weigh more than 70 g.

b. Standardise: $\frac{55 - 64}{9} = -1$ and $\frac{73 - 64}{9} = 1$. 55 and 73 are one standard

deviation either side of the mean, so $P = P(-1 < Z < 1) = 0.683$. So about 68% of tomatoes weigh between 55 g and 73 g.

c. Use the inverse Normal with area 0.2 and rearrange $k = \mu + z\sigma$. $z = -0.8416$, so $k = 64 - 0.8416 \times 9 = 56.4$ g. So 20% of tomatoes weigh less than 56.4 g.

Answer to Question 4 (4 marks)

Mark scheme: a) 1 mark per equation; b) 1 mark for σ, 1 mark for μ.

a. Change each statement into an area to the left: $P(X > 30) = 0.4$ means $P(X < 30) = 0.6$. Then use the inverse Normal. $P(X < 20) = 0.15$: $\frac{20 - \mu}{\sigma} = -1.0364$. $P(X < 30) = 0.6$: $\frac{30 - \mu}{\sigma} = 0.2533$. These are two linear equations in μ and σ.

b. Subtract the equations to eliminate μ, then substitute back. Subtracting: $\frac{10}{\sigma} = 1.2897$, so $\sigma = 7.75$. Then $\mu = 20 + 1.0364 \times 7.75 = 28.0$. So $\mu \approx 28.0$ and $\sigma \approx 7.75$.

Answer to Question 5 (3 marks)

Method: a binomial $B(n, p)$ with large n and p not too close to 0 or 1 can be approximated by $N(np, np(1-p))$. Because X takes whole numbers, the value 40 corresponds to the interval from 39.5 to 40.5, so $X \leq 40$ becomes $Y < 40.5$.

$np = 45$ and $np(1-p) = 31.5$, so $Y \sim N(45, 31.5)$ (1). $P(X \leq 40) \approx P(Y < 40.5)$ (1) $= P\left(Z < \frac{40.5 - 45}{\sqrt{31.5}}\right) = P(Z < -0.802) = 0.211$ (1). (The exact value is 0.2126.)

Answer: $P(X \leq 40) \approx 0.211$, which is very close to the exact binomial value.