Worksheet - Conditional probability and the Normal distribution

Year: Year 13 · 22

Show your working.

Question 1

Question 1 – Conditional probability from the formula *(Section A · Skills practice)*

Events A and B satisfy $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$.

a. Find $P(A \mid B)$ and $P(B \mid A)$. [2 marks]
b. Find $P(A \cup B)$. [1 mark]
c. State, with a reason, whether A and B are independent. [2 marks]

Question 2

Question 2 – Spotting dependence *(Section A · Skills practice)*

For two events $P(X) = 0.4$, $P(Y) = 0.35$ and $P(X \cup Y) = 0.61$.

a. Find $P(X \cap Y)$. [2 marks]
b. Find $P(X \mid Y)$ and compare it with $P(X)$. [2 marks]
c. Hence decide whether X and Y are independent. [1 mark]

Question 3

Question 3 – Counters without replacement *(Section A · Skills practice)*

A bag contains 6 red and 4 yellow counters. Two counters are taken at random without replacement.

a. Draw a tree diagram with all probabilities. [3 marks]
b. Find the probability that both counters are red. [1 mark]
c. Find the probability that the counters are different colours. [2 marks]
d. Find the probability that the second counter is red given that the first was yellow. [1 mark]

Question 4

Question 4 – Normal probabilities *(Section A · Skills practice)*

$X \sim N(120, 15^2)$. Use your calculator to find, to 3 significant figures:

a. $P(X$ [1 mark]
b. $P(X > 150)$ [1 mark]

c. $P(105$ [2 marks]
d. Explain why $P(X = 120) = 0$. [1 mark]

Question 5

Question 5 – The inverse Normal *(Section A · Skills practice)*

$X \sim N(120, 15^2)$. Find the value of a such that:

a. $P(X$ [2 marks]
b. $P(X > a) = 0.9$ [2 marks]
c. $P(X$ [2 marks]

Question 6

Question 6 – One unknown parameter *(Section A · Skills practice)*

$X \sim N(\mu, 8^2)$ and $P(X$.

a. Find the z-value for which $P(Z$. [1 mark]
b. Find μ. [3 marks]
c. Hence find $P(X > 50)$. [2 marks]

Question 7

Question 7 – The school bus *(Section B · Real-life practice)*

The morning bus is on time with probability 0.8. If the bus is on time, Jaya arrives at school before the bell with probability 0.95. If the bus is late, she arrives before the bell with probability 0.4.

a. Draw a tree diagram for this information. [3 marks]
b. Find the probability that Jaya arrives before the bell. [2 marks]
c. Given that Jaya arrives before the bell, find the probability that the bus was on time. [3 marks]
d. Over a 5-day week, the bus is on time each day independently. Find the probability that it is on time on all 5 days. [2 marks]

Question 8

Question 8 – Canteen choices *(Section B · Real-life practice)*

A survey of 120 students asked whether they usually buy lunch in the canteen or bring a packed lunch.

	Canteen	Packed lunch	Total
Year 12	28	32	60
Year 13	42	18	60

	Canteen	Packed lunch	Total
Total	70	50	120

a. A student is chosen at random. Find $P(\text{canteen})$. [1 mark]
b. Find $P(\text{Year 13} \mid \text{canteen})$. [2 marks]
c. Find $P(\text{canteen} \mid \text{Year 12})$. [2 marks]
d. Are the events "canteen" and "Year 12" independent? Justify your answer. [3 marks]

Question 9

Question 9 – Rain and umbrellas *(Section B · Real-life practice)*

In a seaside town, the probability that it rains on a given day in July is 0.3. When it rains, Marcus takes an umbrella with probability 0.9. When it does not rain, he takes one with probability 0.2.

a. Find the probability that Marcus takes an umbrella. [2 marks]
b. Given that Marcus takes an umbrella, find the probability that it rained. [2 marks]
c. Given that Marcus does not take an umbrella, find the probability that it rained. [3 marks]
d. Comment on whether taking an umbrella is a good predictor of rain. [1 mark]

Question 10

Question 10 – Heights of sixth-form students *(Section B · Real-life practice)*

The heights of Year 13 students in a school are modelled by $H \sim N(172, 7^2)$ centimetres.

a. Find the proportion of students taller than 185 cm. [2 marks]
b. Find the proportion between 165 cm and 180 cm. [2 marks]
c. The tallest 5% of students are invited to try out for the basketball team. Find the minimum height for an invitation. [2 marks]
d. The school has 240 students in Year 13. Estimate how many are invited. [1 mark]

Question 11

Question 11 – Tea bags *(Section B · Real-life practice)*

A factory fills tea bags so that the mass of tea is $T \sim N(3.00, 0.08^2)$ grams. A bag is rejected if it contains less than 2.9 g.

a. Find the probability that a randomly chosen bag is rejected. [2 marks]
b. A box holds 5 bags. Find the probability that none is rejected. [2 marks]
c. The factory wishes to reduce the reject rate to 1%. If the standard deviation stays the same, find the new mean. [3 marks]

Question 12

Question 12 – Medical screening *(Section B · Real-life practice)*

2% of a population has a certain condition. A screening test gives a positive result for 90% of people who have the condition and for 7% of people who do not.

a. Find the probability that a randomly chosen person tests positive. [2 marks]
b. Given a positive result, find the probability that the person has the condition. [2 marks]
c. Given a negative result, find the probability that the person does not have the condition. [2 marks]
d. A newspaper says "a positive result means you almost certainly have the condition". Use your answers to criticise the statement. [2 marks]

Question 13

Question 13 – Penalty shoot-out *(Section B · Real-life practice)*

A football player scores a penalty with probability 0.8 if the previous penalty she took was scored and 0.6 if it was missed. She scores her first penalty with probability 0.8.

a. Find the probability that she scores her first two penalties. [1 mark]
b. Find the probability that she scores exactly one of her first two. [3 marks]
c. Given that she scores exactly one of the first two, find the probability that it was the first. [2 marks]
d. Find the probability that she scores her third penalty. [3 marks]

Question 14

Question 14 – Eggs *(Section B · Real-life practice)*

The mass of a hen\'s egg is modelled by $M \sim N(60, 5^2)$ grams. Eggs are graded: small under 53 g, medium from 53 g to 63 g, large over 63 g.

a. Find the proportion of eggs in each grade. [4 marks]
b. A tray holds 6 eggs. Find the probability that all are medium. [2 marks]
c. The farmer wants the smallest 10% to be rejected. Find the cut-off mass. [2 marks]

Question 15

Question 15 – The three-machine factory *(Section C · Problem solving)*

A factory makes smartphone cases on three machines. The table shows the share of production made by each machine and the percentage of its cases that are defective. A case is chosen at random from a day\'s production.

Machine	Share of production	Defective cases
A	50%	3%
B	30%	4%
C	20%	6%

a. Draw a tree diagram. [3 marks]
b. Show that the probability that the case is defective is 0.039. [3 marks]
c. The case is defective. Find the probability that it came from machine A, and from machine C. [4 marks]
d. The manager says "machine C causes the most defective cases because it has the highest defect rate". Use your answers to comment. [2 marks]

Question 16

Question 16 – Journey times *(Section C · Problem solving)*

The time taken for a lorry to drive between two depots is modelled by $T \sim N(\mu, \sigma^2)$ minutes. The depot manager knows that 10% of journeys take less than 35 minutes and 15% take more than 50 minutes.

a. Show that $\dfrac{35-\mu}{\sigma} = -1.2816$ and find a second equation for μ and σ. [3 marks]
b. Find μ and σ. [4 marks]
c. Find the probability that a journey takes between 30 and 45 minutes. [2 marks]
d. The lorry must set off so that it arrives on time on 99% of days. How many minutes should be allowed? [2 marks]

Question 17

Question 17 – The sports club Venn diagram *(Section C · Problem solving)*

A sports club has 60 members. 32 play badminton (B), 28 play squash (S) and 12 play both. A member is chosen at random.

a. Draw a Venn diagram showing the number of members in each region. [3 marks]
b. Find $P(B \mid S)$ and $P(S \mid B)$. [3 marks]
c. Find $P(\text{neither})$ and $P(\text{exactly one sport})$. [3 marks]
d. Are B and S independent? Show your working. [3 marks]

Question 18

Question 18 – Overbooking a ferry *(Section C · Problem solving)*

A ferry has 190 places for foot passengers. The company sells 200 tickets for each sailing. Each ticket-holder independently turns up with probability 0.9. Let X be the number of ticket-holders who turn up.

a. State the distribution of X and show that it may be approximated by $Y \sim N(180, 18)$. [3 marks]
b. Using a continuity correction, estimate the probability that more than 190 turn up, so that someone is refused. [4 marks]
c. The exact value found using the binomial distribution is 0.0035. Compare it with your estimate and explain why they differ. [2 marks]
d. The company wants the probability of refusing someone to be below 0.5%. Is selling 200 tickets acceptable? [1 mark]

Question 19

Question 19 – Is the Normal model sensible? *(Section C · Problem solving)*

The table summarises the reaction times (in milliseconds) of 200 students tested during a science fair. The mean is 255 ms and the standard deviation is 40 ms. A teacher models the times by $R \sim N(255, 40^2)$.

Time (ms)	under 215	215 to 255	255 to 295	over 295
Frequency	31	69	64	36

a. Use the model to find the expected frequency in each class. [4 marks]
b. Compare with the observed frequencies and comment on whether the Normal model is a good fit. [2 marks]
c. Under the model, find the reaction time exceeded by only 1% of students. [2 marks]
d. Explain why a Normal model would be unsuitable for the time a customer waits in a queue. [1 mark]

Question 20

Question 20 – Normal and conditional together *(Section C · Problem solving)*

The lifetime of a type of battery is modelled by $L \sim N(500, 50^2)$ hours. Batteries lasting under 420 hours are classed as faulty. Quality control rejects a faulty battery with probability 0.95 and wrongly rejects a good one with probability 0.02.

a. Find the probability that a battery is faulty. [2 marks]
b. Find the probability that a battery is rejected. [3 marks]
c. Given that a battery is rejected, find the probability that it really was faulty. [3 marks]
d. Find the probability that a faulty battery is not rejected. [1 mark]

Question 21

Question 21 – Challenge: independent or not? *(Section D · Challenge)*

A and B are events with $P(A) = 0.5$ and $P(A \cup B) = 0.8$.

a. Given that A and B are independent, find $P(B)$. [3 marks]
b. Given instead that $P(B \mid A) = 0.2$, find $P(B)$. [3 marks]
c. Given instead that A and B are mutually exclusive, find $P(B)$, and show that A and B cannot then be independent. [3 marks]

Question 22

Question 22 – Challenge: the optimal threshold *(Section D · Challenge)*

The marks of Year 13 students in a mock exam are modelled by $M \sim N(\mu, \sigma^2)$. 20% of students score below 38 and 10% score above 78.

a. Find μ and σ. [5 marks]
b. The pass mark is set so that exactly 70% of students pass. Find it. [2 marks]
c. Three students are chosen at random. Find the probability that exactly two of them pass. [3 marks]