Test answers - Vectors in two dimensions

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (4 marks)

a. Multiply each vector by its scalar, then subtract component by component. $12\mathbf{i} - 9\mathbf{j} - 2\mathbf{i} - 4\mathbf{j} = 10\mathbf{i} - 13\mathbf{j}$ (2) So $3\mathbf{a} - 2\mathbf{b} = 10\mathbf{i} - 13\mathbf{j}$.

b. Find $\mathbf{a} + 2\mathbf{b}$ first, then use Pythagoras for the magnitude. $\mathbf{a} + 2\mathbf{b} = 6\mathbf{i} + \mathbf{j}$ (1), magnitude $\sqrt{37} = 6.08$ (1) So $|\mathbf{a} + 2\mathbf{b}| = \sqrt{37} \approx 6.08$.

Check (b) with components: $(4 + 2)\mathbf{i} + (-3 + 4)\mathbf{j} = 6\mathbf{i} + \mathbf{j}$ ✓. Writing $\sqrt{6^2 + 1}$ as 7 is a frequent arithmetic slip.

Answer to Question 2 (4 marks)

a. Parallel vectors are scalar multiples, so the ratios of corresponding components are equal. $\frac{k+2}{2} = \frac{9}{3} = 3$ (1), $k = 4$ (1) Check: $\mathbf{p} = 6\mathbf{i} + 9\mathbf{j} = 3(2\mathbf{i} + 3\mathbf{j})$ ✓.

b. If $\mathbf{p} = k\mathbf{q}$ with $k > 0$, then $|\mathbf{p}| = k|\mathbf{q}|$. $\mathbf{p} = 3\mathbf{q}$ (1) so the ratio is $3 : 1$ (1) So $\mathbf{p}$ is three times as long as $\mathbf{q}$.

Equivalently $3(k + 2) = 2 \times 9$, giving $k + 2 = 6$.

Answer to Question 3 (4 marks)

a.  ; its magnitude comes from Pythagoras. $6\mathbf{i} - 6\mathbf{j}$ (1), $6\sqrt{2}$ (1) So .

b. Ratio $2 : 1$ means P is $\frac{2}{3}$ of the way from A to B.  (1),  (1) So P is the point $(5, 0)$.

Alternatively  ✓. Taking $\frac{1}{3}$ of  gives $(3, 2)$, the point dividing AB in the ratio $1 : 2$.

Answer to Question 4 (4 marks)

a. For a bearing θ, the east component is $v \sin\theta$ and the north component is $v \cos\theta$.

$20\sin 150^\circ = 10$ and $20\cos 150^\circ = -17.32$, so $10\mathbf{i} - 17.3\mathbf{j}$ (2) The ship heads south-east, so the $\mathbf{j}$ component is negative.

b. With constant velocity, displacement = velocity × time. $30\mathbf{i} - 52.0\mathbf{j}$ (1) After 3 hours the ship is 30 km east and 52 km south of its start.

c. Read off the $\mathbf{i}$ component of the position vector. 30 km (1) The ship is 30 km east of its starting point.

Answer to Question 5 (4 marks)

Write and , then find and compare it with .

(1). (1) (1). A scalar multiple, so parallel and half the length (1).

This is the midpoint theorem: the line joining the midpoints of two sides of a triangle is parallel to the third side and half as long.