Test answers - Trigonometry

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (4 marks)

a. Two sides and the included angle are known, so use the cosine rule. $BC^2 = 49 + 100 - 140\cos 58^\circ = 149 - 74.19 = 74.81$ (1), $BC = 8.65$ cm (1) So $BC \approx 8.65$ cm.

b. Use area $= \frac{1}{2}bc\sin A$ with the two given sides and the angle between them. Area $= \frac{1}{2} \times 7 \times 10\sin 58^\circ$ (1) $= 29.7$ cm^2 (1) The area is about 29.7 cm^2.

Make sure the calculator is in degree mode; in radian mode $\cos 58$ gives a nonsense length. Keep BC^2 unrounded before taking the square root.

Answer to Question 2 (4 marks)

a. In radians, arc length $= r\theta$. $9 \times 0.8 = 7.2$ cm (1) The arc is 7.2 cm long.

b. In radians, sector area $= \frac{1}{2}r^2\theta$. $\frac{1}{2} \times 81 \times 0.8 = 32.4$ cm^2 (1) The sector has area 32.4 cm^2.

c. Segment $=$ sector $-$ triangle $= \frac{1}{2}r^2(\theta - \sin\theta)$, with the calculator in radians. Segment $= \frac{1}{2} \times 81(0.8 - \sin 0.8)$ (1) $= 40.5 \times 0.0827 = 3.35$ cm^2 (1) The segment has area about 3.35 cm^2.

Check: the triangle has area $\frac{1}{2} \times 81\sin 0.8 \approx 29.05$ cm^2, and $32.4 - 29.05 = 3.35$ ✓.

Answer to Question 3 (4 marks)

Start with the left-hand side and add the fractions over the common denominator $\sin\theta(1 + \cos\theta)$; then use $\sin^2\theta + \cos^2\theta = 1$.

$$\text{LHS} = \frac{\sin^2\theta + (1 + \cos\theta)^2}{\sin\theta(1 + \cos\theta)} \text{ (1)} = \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1 + \cos\theta)} \text{ (1)} = \frac{2 + 2\cos\theta}{\sin\theta(1 + \cos\theta)} \text{ (1)}$$

$$= \frac{2(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)} = \frac{2}{\sin\theta} \text{ (1).}$$

The factor $(1 + \cos\theta)$ cancels, which gives the right-hand side and proves the identity.

The working should go from one side to the other, not change both sides at once. A useful check: at $\theta = 90^\circ$ the left-hand side is $1 + 1 = 2$ and the right-hand side is $\frac{2}{1} = 2$ ✓.

Answer to Question 4 (4 marks)

Use $\sin^2\theta = 1 - \cos^2\theta$ to obtain a quadratic in $\cos\theta$, then factorise.

$2(1-\cos^2\theta) = 1+\cos\theta$ (1), so $2\cos^2\theta+\cos\theta-1=0$ (1), $(2\cos\theta-1)(\cos\theta+1)=0$ (1). $\cos\theta=\frac{1}{2}$: $60°, 300°$; $\cos\theta=-1$: $180°$ (1).

So the solutions are $\theta = 60°$, $180°$ and $300°$. Check $180°$: $2\times 0 = 1+(-1)$ ✓.

Note that $\cos\theta=-1$ has only one solution in this interval.

Dividing both sides by $\cos\theta+1$ at the start would lose the solution $180°$. Check $60°$: $2\times\frac{3}{4} = 1.5 = 1+\frac{1}{2}$ ✓.

Answer to Question 5 (4 marks)

a. The bearing of H from A is the back-bearing $065°+180° = 245°$; the boat then heads on $140°$. Back bearing at A is $245°$; $245°-140° = 105°$ (1) So angle $HAB = 105°$.

b. Use the cosine rule with $HA = 12$, $AB = 9$ and the included angle $105°$. $HB^2 = 144+81-2\times 12\times 9\cos 105° = 225+55.9 = 280.9$ (1), $HB = 16.8$ km (1) So $HB \approx 16.8$ km.

c. Find angle AHB with the sine rule, then add it to the bearing of A from H. $\sin H = \dfrac{9\sin 105°}{16.76} = 0.5187$, $H = 31.2°$, bearing $065°+31° = 096°$ (1) The bearing of B from H is $096°$.