Worksheet - Trigonometry

Year: Year 12 · 20

Show your working.

Question 1

Question 1 – The sine rule *(Section A · Skills practice)*

In triangle ABC, angle $A = 48^\circ$, angle $B = 65^\circ$ and side $a = BC = 12$ cm.

a. Find angle C. [1 mark]
b. Use the sine rule to find the length b to 3 significant figures. [3 marks]
c. Find the length c to 3 significant figures. [3 marks]

Question 2

Question 2 – The cosine rule and the area of a triangle *(Section A · Skills practice)*

Use the cosine rule and area formula.

a. In triangle ABC, $a = 7$ cm, $b = 9$ cm and angle $C = 52^\circ$. Find c to 3 significant figures. [3 marks]
b. Triangle PQR has sides 8 cm, 11 cm and 14 cm. Find the smallest angle to 1 decimal place. [4 marks]
c. A triangle has sides 6.4 cm and 9.1 cm with an included angle of 38°. Find its area to 3 significant figures. [2 marks]
d. A triangle with sides 8 cm and 10 cm has area 20 cm^2. Find the two possible sizes of the included angle. [3 marks]

Question 3

Question 3 – Radians, arcs and sectors *(Section A · Skills practice)*

Answer these questions using radians.

a. Convert 135° to radians and $\dfrac{5\pi}{6}$ to degrees. [2 marks]
b. A sector has radius 8 cm and angle 1.2 radians. Find the arc length and the area of the sector. [3 marks]
c. A sector has radius 6 cm and arc length 15 cm. Find its angle and its area. [3 marks]
d. A circle has radius 10 cm. A chord subtends an angle of 1.2 radians at the centre. Find the area of the segment to 2 decimal places. [3 marks]

Question 4

Question 4 – Exact values *(Section A · Skills practice)*

Without a calculator, find the exact value of each expression.

a. $\sin 60^\circ \cos 30^\circ - \tan 45^\circ$ [3 marks]
b. $\dfrac{\tan \frac{\pi}{3}}{\sin \frac{\pi}{4}}$ [3 marks]
c. $\cos 150^\circ$, $\sin 210^\circ$ and $\tan 135^\circ$ [3 marks]

Question 5

Question 5 – Trigonometric identities *(Section A · Skills practice)*

Use $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta \equiv 1$.

a. Prove that $\dfrac{1-\cos^2\theta}{\sin\theta\cos\theta} \equiv \tan\theta$. [3 marks]
b. Prove that $\tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{1}{\sin\theta\cos\theta}$. [4 marks]
c. Given that $\sin\theta = \dfrac{3}{5}$ and θ is acute, find the exact values of $\cos\theta$ and $\tan\theta$. [3 marks]

Question 6

Question 6 – Solving trigonometric equations *(Section A · Skills practice)*

Solve each equation for $0^\circ \le \theta \le 360^\circ$. Give answers to 1 decimal place where necessary.

a. $\sin\theta = 0.6$ [2 marks]
b. $2\cos\theta = -1$ [2 marks]
c. $\tan\theta = -2$ [2 marks]
d. $\sin 2\theta = 0.5$ [3 marks]

Question 7

Question 7 – Surveying a lake *(Section B · Real-life practice)*

A surveyor stands at a point O on one side of a lake. The points A and B are on opposite shores. $OA = 120$ m, $OB = 95$ m and angle $AOB = 70^\circ$.

a. Find the width AB of the lake to the nearest metre. [3 marks]
b. Find the area of triangle OAB to the nearest m^2. [2 marks]
c. The triangle of land OAB is for sale at £14.50 per m^2. Work out the price to the nearest £100. [2 marks]

d. Find angle OAB to 1 decimal place. [3 marks]

Question 8

Question 8 – A ship on a bearing *(Section B · Real-life practice)*

A ship leaves port P and sails 20 km on a bearing of 040° to a buoy Q. It then sails 30 km on a bearing of 110° to a lighthouse R.

a. Show that angle $PQR = 110^\circ$. [2 marks]
b. Find the distance PR to 1 decimal place. [3 marks]
c. Find the bearing of R from P to the nearest degree. [4 marks]
d. The ship returns directly from R to P at 18 km/h. How long does the return take, to the nearest minute? [2 marks]

Question 9

Question 9 – The height of a tower *(Section B · Real-life practice)*

Two observers stand on level ground in line with the base of a tall tower. Observer A is further away. The observers are 40 m apart ($AB = 40$ m). The angle of elevation of the top T of the tower is 32° from A and 41° from B.

a. Show that angle $ATB = 9^\circ$. [2 marks]
b. Use the sine rule in triangle ATB to find AT to 1 decimal place. [3 marks]
c. Find the height of the tower to the nearest metre. [2 marks]
d. How far is B from the base of the tower, to the nearest metre? [3 marks]

Question 10

Question 10 – Tides in the harbour *(Section B · Real-life practice)*

The depth of water in a harbour entrance is modelled by $h = 4 + 2\sin(30t^\circ)$ metres, where t is the number of hours after midnight, $0 \le t \le 12$.

a. Find the depth at 3 am and at 9 am. [2 marks]
b. Find the times when the depth is 5 m. [3 marks]
c. A ferry needs at least 4.5 m of water. Between which times, to the nearest minute, can it enter during this period? [4 marks]
d. State the period of the model. Explain what it means. [2 marks]

Question 11

Question 11 – A sector flower bed *(Section B · Real-life practice)*

A garden designer plans a flower bed in the shape of a sector of a circle with radius 6 m and angle $\dfrac{2\pi}{3}$ radians. Fencing costs £6.50 per metre and turf costs £4.20 per m^2.

a. Find the area of the bed in terms of π and as a decimal. [3 marks]
b. Find the perimeter (the arc and two straight edges). [3 marks]
c. Work out the cost of fencing all the way round. [2 marks]
d. Work out the cost of turf for the bed. [2 marks]

Question 12

Question 12 – The stained-glass segment *(Section B · Real-life practice)*

A window has a circular segment of coloured glass cut from a circle of radius 10 cm. The chord AB subtends an angle of 1.2 radians at the centre O.

a. Find the length of the chord AB to 2 decimal places. [3 marks]
b. Find the area of triangle OAB. [2 marks]
c. Find the area of the segment. [3 marks]
d. The glass costs £0.12 per cm^2. Find the cost of the segment. [2 marks]

Question 13

Question 13 – A farmer's triangular field *(Section B · Real-life practice)*

A triangular field has sides of 120 m, 150 m and 200 m. Seed costs £0.45 per m^2.

a. Find the largest angle of the triangle. [3 marks]
b. Find the area of the field. [2 marks]
c. Check your area using Heron\'s formula. [3 marks]
d. Find the cost of the seed to the nearest £10. [2 marks]

Question 14

Question 14 – A quadratic in sine *(Section B · Real-life practice)*

A light show pattern is described by $y = 2\sin^2 x + \sin x - 1$, where x is in degrees.

a. Factorise the quadratic expression in $\sin x$. [2 marks]
b. Solve $y = 0$ for $0^\circ \le x \le 360^\circ$. [4 marks]
c. Find the maximum and minimum values of y. [3 marks]

Question 15

Question 15 – Surveying a quadrilateral plot *(Section C · Problem solving)*

A plot of land $ABCD$ is divided by the diagonal AC into two triangles. The measurements are in the table.

Measurement	Value
AB	8 m
BC	6 m

Measurement	Value
angle ABC	75°
AD	9 m
CD	5 m

a. Find AC to 2 decimal places. [3 marks]
b. Find angle ADC to 1 decimal place. [4 marks]
c. Find the total area of $ABCD$. [4 marks]
d. The owner wants a fence along AC costing £11 per metre. Find the cost. [1 mark]

Question 16

Question 16 – Identities and equations *(Section C · Problem solving)*

Use identities to solve each problem.

a. Solve $2\cos^2\theta + 5\sin\theta - 4 = 0$ for $0^\circ \le \theta \le 360^\circ$. [5 marks]
b. Solve $\tan\theta = 2\sin\theta$ for $0^\circ \le \theta \le 360^\circ$. [5 marks]
c. Solve $\cos(\theta + 40^\circ) = -\frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$. [4 marks]

Question 17

Question 17 – Largest sector *(Section C · Problem solving)*

A sector of a circle has perimeter 20 cm. The radius is r cm and the angle is θ radians. Here are some possibilities.

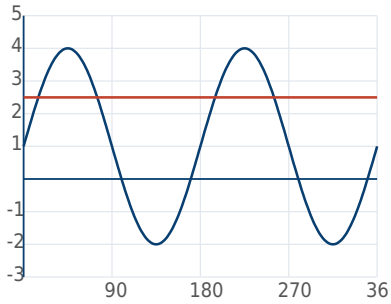
r (cm)	2	4	5	8
θ (rad)	8	3	2	0.5
Area (cm^2)	16	24	25	16

a. Show that $\theta = \dfrac{20 - 2r}{r}$. [2 marks]
b. Check the table for $r = 4$ and $r = 8$. [2 marks]
c. Show that the area is $A = r(10 - r)$. [2 marks]
d. Complete the square to find the greatest possible area and the matching value of θ. [4 marks]

Question 18

Question 18 – Graph of a trigonometric function *(Section C · Problem solving)*

The graph shows $y = 3\sin 2x + 1$ for $0^\circ \le x \le 360^\circ$.



a. State the maximum and minimum values of y. [2 marks]
b. State the period. [1 mark]
c. Solve $3\sin 2x + 1 = 2.5$ for $0^\circ \leq x \leq 360^\circ$. [4 marks]
d. Describe the transformations that map $y = \sin x$ to $y = 3\sin 2x + 1$. [3 marks]

Question 19

Question 19 – Proving and using an identity *(Section C · Problem solving)*

Work with identities to solve a problem.

a. Prove that $\dfrac{\cos\theta}{1-\sin\theta} \equiv \dfrac{1+\sin\theta}{\cos\theta}$. [4 marks]
b. Prove that $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} \equiv \dfrac{2}{\cos^2\theta}$. [3 marks]
c. Hence solve $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 8$ for $0^\circ \leq \theta \leq 360^\circ$. [4 marks]

Question 20

Question 20 – Challenge: the ambiguous case *(Section D · Challenge)*

In triangle ABC, $a = 7$ cm, $b = 10$ cm and angle $A = 40^\circ$.

a. Show that there are two possible triangles and find the two possible values of angle B. [4 marks]
b. For each triangle find the length c. [4 marks]
c. For fixed $b = 10$ and $A = 40^\circ$, find the range of values of a for which two triangles are possible. [3 marks]