Test answers - Straight lines and circles

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (4 marks)

a. Find the gradient, then use $y - y_1 = m(x - x_1)$ with $A(1, 3)$. Gradient $= \dfrac{11 - 3}{5 - 1} = 2$ (1). $y - 3 = 2(x - 1)$ so $y = 2x + 1$ (1) Check with B: $2 \times 5 + 1 = 11$ ✓.

b. The perpendicular bisector passes through the midpoint of AB and has gradient $-1 \div 2 = -\frac{1}{2}$. Midpoint $(3, 7)$ (1). Gradient $-\frac{1}{2}$: $y - 7 = -\frac{1}{2}(x - 3)$, so $y = -\frac{1}{2}x + \frac{17}{2}$ (1) Check: the midpoint $(3, 7)$ satisfies $-\frac{3}{2} + \frac{17}{2} = 7$ ✓.

The perpendicular bisector can also be written $x + 2y = 17$, which is often the neater form to quote.

Answer to Question 2 (4 marks)

a. Complete the square in x and in y, then read off the centre and the radius. $(x - 5)^2 + (y + 2)^2 = 25 + 4 - 13 = 16$ (1). Centre $(5, -2)$ (1), radius 4 (1) So the circle has centre $(5, -2)$ and radius 4.

b. Compare the squared distance from the centre with $r^2 = 16$. $(1 - 5)^2 + (-1 + 2)^2 = 17 > 16$ so it lies outside (1) The point is just outside the circle.

A sign slip giving the centre $(-5, 2)$ is the most common error.

Answer to Question 3 (4 marks)

a. Substitute P into the left-hand side. $(7 - 3)^2 + (1 + 2)^2 = 16 + 9 = 25$ (1) So P lies on the circle.

b. The tangent is perpendicular to the radius from the centre $(3, -2)$ to P. Radius gradient $\frac{3}{4}$ (1). Tangent gradient $-\frac{4}{3}$ (1). $y - 1 = -\frac{4}{3}(x - 7)$, i.e. $4x + 3y = 31$ (1) Check: $4 \times 7 + 3 \times 1 = 31$ ✓.

Radius gradient: $\dfrac{1 - (-2)}{7 - 3} = \dfrac{3}{4}$. The product of the gradients is $\frac{3}{4} \times (-\frac{4}{3}) = -1$ ✓, so the lines are perpendicular. Using the radius gradient itself gives the normal, not the tangent.

Answer to Question 4 (3 marks)

Substitute the line into the circle. A tangent meets the circle at exactly one point, so

the resulting quadratic has equal roots and its discriminant is zero.

$x^2 + (x+k)^2 = 8 \Rightarrow 2x^2 + 2kx + k^2 - 8 = 0$ (1). Discriminant $4k^2 - 8(k^2 - 8) = 64 - 4k^2 = 0$ (1), so $k = \pm 4$ (1).

So the lines $y = x + 4$ and $y = x - 4$ are the two tangents; they touch the circle at $(-2, 2)$ and $(2, -2)$.

Check: the distance from the origin to $x - y + 4 = 0$ is $\frac{4}{\sqrt{2}} = 2\sqrt{2} = \sqrt{8}$, the radius ✓.

Answer to Question 5 (5 marks)

a. The centre is the midpoint of the diameter and the radius is half its length. Centre $(6, 6)$ (1). $AB = \sqrt{64 + 36} = 10$ so radius 5 (1). $(x-6)^2 + (y-6)^2 = 25$ (1)

b. Substitute C into the equation of the circle. $(9-6)^2 + (10-6)^2 = 9 + 16 = 25$ (1) So the kiosk lies on the boundary.

c. Perpendicular lines have gradients that multiply to -1. Gradient $AC = \frac{7}{7} = 1$ and gradient $BC = \frac{1}{-1} = -1$; product -1 (1) So angle $ACB = 90^\circ$, as expected for the angle in a semicircle.