Worksheet - Straight lines and circles

Year: Year 12 · 20

Show your working.

Question 1

Question 1 – Gradient, midpoint and length *(Section A · Skills practice)*

The points A and B have coordinates $A(2, -1)$ and $B(8, 7)$.

a. Find the gradient of AB. [1 mark]
b. Find the equation of the line AB in the form $ax + by + c = 0$ where a, b, c are integers. [3 marks]
c. Find the midpoint M of AB. [1 mark]
d. Find the length AB. [2 marks]

Question 2

Question 2 – Parallel and perpendicular lines *(Section A · Skills practice)*

The line L has equation $3x + 4y = 12$.

a. Find the gradient of L. [1 mark]
b. Find the equation of the line parallel to L through $(4, -2)$. [2 marks]
c. Find the equation of the line perpendicular to L through $(0, 5)$. [2 marks]
d. Find the coordinates of the foot of the perpendicular from $(0, 5)$ to L and the distance from $(0, 5)$ to L. [4 marks]

Question 3

Question 3 – Equation of a circle *(Section A · Skills practice)*

A circle has centre $C(3, -2)$ and radius 5.

a. Write down its equation in the form $(x - a)^2 + (y - b)^2 = r^2$. [2 marks]
b. Expand to the form $x^2 + y^2 + px + qy + s = 0$. [2 marks]
c. Show that the points $(6, 2)$ and $(0, 2)$ lie on the circle. [2 marks]
d. Determine whether $(7, 1)$ lies inside, outside or on the circle. [2 marks]

Question 4

Question 4 – Centre and radius from the general equation *(Section A · Skills practice)*

Find the centre and radius of each circle.

a. $x^2 + y^2 - 8x + 6y - 11 = 0$ [3 marks]
b. $x^2 + y^2 + 2x - 4y - 4 = 0$ [3 marks]
c. Find the distance between the two centres and decide whether the circles intersect. [3 marks]

Question 5

Question 5 – The angle in a semicircle *(Section A · Skills practice)*

$A(1, 2)$ and $B(7, 10)$ are the ends of a diameter of a circle.

a. Find the centre and the radius. [3 marks]
b. Write down the equation of the circle. [1 mark]
c. Show that $C(8, 9)$ lies on the circle. [2 marks]
d. Use gradients to show that angle ACB is a right angle and name the circle theorem. [3 marks]

Question 6

Question 6 – Tangent at a point *(Section A · Skills practice)*

The circle C has equation $(x - 1)^2 + (y + 2)^2 = 25$.

a. Show that $P(4, 2)$ lies on C. [1 mark]
b. Find the gradient of the radius to P. [1 mark]
c. Find the equation of the tangent at P in the form $ax + by = c$. [3 marks]
d. The tangent meets the x-axis at T. Find the coordinates of T and the area of the triangle formed by the tangent and the two axes. [4 marks]

Question 7

Question 7 – Mobile phone coverage *(Section B · Real-life practice)*

On a map (units in km) a phone mast is at $M(3, 4)$. It gives coverage within 5 km. A straight road is the line $y = x - 2$.

a. Write down the equation of the boundary of the coverage area. [1 mark]
b. Show that the road enters the coverage area where $x^2 - 9x + 10 = 0$. [4 marks]
c. Find the length of road within the coverage area, to 2 decimal places. [3 marks]
d. Find the shortest distance from the mast to the road. [3 marks]

Question 8

Question 8 – A circular pond *(Section B · Real-life practice)*

A garden plan uses metres. A circular pond is the circle $(x - 6)^2 + (y - 5)^2 = 9$. A straight path is the line $3x + 4y = 53$.

a. Write down the centre and radius of the pond. [2 marks]
b. Show that the path is a tangent to the pond. [5 marks]
c. Find the coordinates of the point where the path touches the pond. [3 marks]
d. Find the equation of the other tangent to the pond of the form $3x + 4y = k$. [2 marks]

Question 9

Question 9 – The Ferris wheel *(Section B · Real-life practice)*

A Ferris wheel has its centre 12 m above the ground and a radius of 10 m. Take the origin at ground level directly below the centre, with x horizontal and y the height above the ground.

a. Write down the equation of the circle that the cabins follow. [2 marks]
b. A cabin is 6 m horizontally from the centre. Find its two possible heights. [3 marks]
c. What is the lowest height of a cabin? [1 mark]
d. A cabin is 5 m horizontally from the centre and in the upper half. Find its height to 1 decimal place. [2 marks]

Question 10

Question 10 – A fountain through three posts *(Section B · Real-life practice)*

Three lamp posts stand at $A(0, 0)$, $B(8, 0)$ and $C(0, 6)$ on a plan (metres). A circular fountain must pass through all three posts.

a. Find the equations of the perpendicular bisectors of AB and AC. [3 marks]
b. Hence find the centre of the circle and its radius. [3 marks]
c. Write down the equation of the circle in the form $x^2 + y^2 + px + qy + s = 0$. [3 marks]
d. Which side of the triangle is a diameter? Explain. [2 marks]

Question 11

Question 11 – Fencing parallel to a road *(Section B · Real-life practice)*

On a plan a road passes through $(0, 2)$ and $(6, 5)$. A farmer wants a fence parallel to the road passing through $F(2, 6)$.

a. Find the equation of the road and of the fence. [4 marks]
b. The farmer builds a straight track from F meeting the road at right angles. Find the equation of the track. [2 marks]
c. Find where the track meets the road. [3 marks]
d. Find the shortest distance from the fence to the road, to 2 decimal places. [2 marks]

Question 12

Question 12 – Two Wi-Fi routers *(Section B · Real-life practice)*

Router R_1 covers a circle $(x-2)^2+(y-3)^2=16$ and router R_2 covers the circle $(x-8)^2+(y-3)^2=9$ (units: metres).

a. Find the distance between the routers and show that the coverage areas overlap. [3 marks]
b. Show that the x-coordinate of the intersection points of the boundaries is $\frac{67}{12}$. [3 marks]
c. Find the two y-coordinates to 2 decimal places. [3 marks]
d. Find the length of the common chord to 2 decimal places. [1 mark]

Question 13

Question 13 – Tangent to a lake *(Section B · Real-life practice)*

A circular lake has equation $(x-2)^2+(y-1)^2=13$ on a map (km). A straight track is built to touch the lake at $P(5,3)$.

a. Show that P lies on the lake boundary. [1 mark]
b. Find the equation of the tangent at P. [4 marks]
c. The track runs from the x-axis to the y-axis. Find the coordinates of its ends. [2 marks]
d. Find the length of the track, to 2 decimal places. [2 marks]

Question 14

Question 14 – Meeting point equidistant from two villages *(Section B · Real-life practice)*

Two villages are at $A(1,2)$ and $B(9,6)$. A new bus stop must be the same distance from each village and must also be on the straight road $y=x+2$.

a. Find the midpoint and the gradient of AB. [2 marks]
b. Find the equation of the perpendicular bisector of AB. [3 marks]
c. Find the coordinates of the bus stop. [2 marks]
d. Show that the bus stop is 5 km from each village. [2 marks]

Question 15

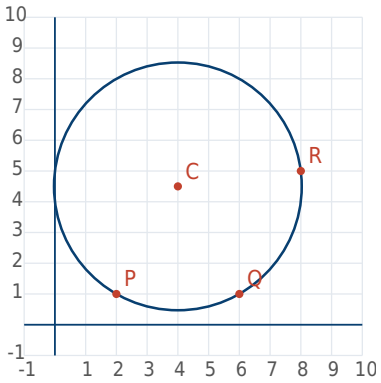
Question 15 – The circle through three points *(Section C · Problem solving)*

A landscape architect wants a circular flower bed passing through the three points $P(2,1)$, $Q(6,1)$ and $R(8,5)$ on a plan (metres).

Point	x	y
P	2	1

Point	x	y
Q	6	1
R	8	5

a. Find the perpendicular bisector of PQ. [2 marks]
b. Find the perpendicular bisector of PR. [3 marks]
c. Find the centre and the exact radius of the circle. [4 marks]
d. Show that R lies on the circle and find the area of the flower bed to 1 decimal place. [3 marks]



Question 16

Question 16 – A line and a circle with a parameter *(Section C · Problem solving)*

The circle C has equation $x^2 + y^2 = 20$. The line L has equation $y = 2x + k$.

a. Show that the x-coordinates of the intersections satisfy $5x^2 + 4kx + k^2 - 20 = 0$. [3 marks]
b. Find the values of k for which L is a tangent to C. [3 marks]
c. State the range of k for which L crosses C at two points. [2 marks]
d. For $k = 5$ find the points of intersection and the length of the chord. [4 marks]

Question 17

Question 17 – Tangent length from an external point *(Section C · Problem solving)*

The circle C has equation $x^2 + y^2 - 6x - 4y - 12 = 0$.

a. Find the centre and radius of C. [3 marks]
b. Show that $P(6, 6)$ lies on C and find the equation of the tangent at P. [4 marks]
c. The tangent meets the x-axis at T. Find the coordinates of T. [2 marks]
d. Show that $TP = 10$. Then show that the length of the other tangent from T to C is also 10. [3 marks]

Question 18

Question 18 – Two overlapping lakes *(Section C · Problem solving)*

Two circular lakes on a map (km) have equations $x^2 + y^2 = 25$ and $(x - 6)^2 + y^2 = 25$.

a. State the centres and radii. [2 marks]
b. Show that the lakes meet where $x = 3$ and find the two points. [4 marks]
c. Find the length of the common chord and show that the chord is perpendicular to the line joining the centres. [3 marks]
d. Show that the common chord passes through the midpoint of the line joining the centres. [2 marks]

Question 19

Question 19 – Right angles in a circle *(Section C · Problem solving)*

The circle $x^2 + y^2 = 25$ has a diameter AB where $A(-5, 0)$ and $B(5, 0)$.

Point	Coordinates
P	$(3, 4)$
Q	$(-4, 3)$
R	$(0, -5)$

a. Show that P, Q and R lie on the circle. [3 marks]
b. For each point work out the gradients of the lines to A and to B and show that they multiply to -1. [6 marks]
c. Let $T(x, y)$ be any point on the circle other than A or B. Prove that the gradients of TA and TB multiply to -1. [4 marks]

Question 20

Question 20 – Challenge: circles touching the axis *(Section D · Challenge)*

A circle has its centre on the line $y = x$ and touches the x-axis. It passes through the point $(8, 4)$.

a. Let the centre be (a, a) with $a > 0$. Explain why the radius is a and write down the equation of the circle. [2 marks]
b. Show that $a^2 - 24a + 80 = 0$ and find the two possible circles. [5 marks]
c. Show that the two circles meet at two points and find the second point of intersection. [5 marks]