Test answers - Simultaneous equations, inequalities and polynomials

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (5 marks)

One equation is linear and one is not, so make y the subject of the linear equation and substitute.

$y = x - 2$ (1). $x(x-2) = 15$ (1), $x^2 - 2x - 15 = 0$ (1), $(x-5)(x+3) = 0$ (1). Solutions: $(5, 3)$ and $(-3, -5)$ (1).

For each x use $y = x - 2$: $x = 5$ gives $y = 3$ and $x = -3$ gives $y = -5$. Check: $5 \times 3 = 15$ ✓ and $(-3)(-5) = 15$ ✓.

Pairing the values wrongly, for example $(5, -5)$, loses the last mark.

Answer to Question 2 (4 marks)

Factorise the quadratic and use a sketch: a U-shaped graph is positive outside its roots. Then solve the linear inequality and combine the two.

$(2x+1)(x-4) \geq 0$ (1), so $x \leq -\frac{1}{2}$ or $x \geq 4$ (1). $3x - 6 < 2x + 5$ gives $x < 11$ (1). Both: $x \leq -\frac{1}{2}$ or $4 \leq x < 11$ (1).

On a number line, the overlap is everything up to $-\frac{1}{2}$ together with the interval from 4 up to (but not including) 11.

The value $x = 11$ is excluded because the linear inequality is strict.

Answer to Question 3 (4 marks)

a. By the factor theorem, $(x-2)$ is a factor if $f(2) = 0$. $f(2) = 8 - 16 + 2 + 6 = 0$ (1) So $(x-2)$ is a factor.
b. Divide $f(x)$ by $(x-2)$, then factorise the quadratic quotient. $f(x) = (x-2)(x^2 - 2x - 3)$ (2) $= (x-2)(x-3)(x+1)$ (1) So the roots of $f(x) = 0$ are -1, 2 and 3. Check: $f(3) = 27 - 36 + 3 + 6 = 0$ ✓ and $f(-1) = -1 - 4 - 1 + 6 = 0$ ✓.

The quotient can also be found by comparing coefficients: $(x-2)(x^2 + bx + c)$ gives $b - 2 = -4$ and $-2c = 6$, so $b = -2$ and $c = -3$.

Answer to Question 4 (3 marks)

By the remainder theorem, the remainder on dividing by $(x-2)$ is $p(2)$, so set $p(2) = 7$.

$p(2) = 16 + 2a - 5 = 7$ (2), so $2a = -4$ and $a = -2$ (1).

Check: $p(2) = 2 \times 8 - 2 \times 2 - 5 = 7$ ✓. So $a = -2$.

The remainder theorem avoids long division: substituting $x = 2$ gives the remainder directly.

A common error is substituting $x = -2$: the divisor $(x - 2)$ is zero when $x = 2$. Setting $p(2) = 0$ instead of 7 confuses the remainder theorem with the factor theorem and gives the wrong value $a = -5.5$.

Answer to Question 5 (4 marks)

a. The length is $w + 3$, and area = width × length. $w(w + 3) = 54$ so $w^2 + 3w - 54 = 0$ (1) This is the required equation.

b. Factorise and reject the negative root, since a width cannot be negative. $(w + 9)(w - 6) = 0$ (1), width 6 m and length 9 m (1) Check: $6 \times 9 = 54$ ✓.

c. The perimeter is $2(6 + 9) = 30$ m; multiply by the cost per metre. Perimeter $= 30$ m, cost $= 30 \times 4.5 =$ £135 (1) Edging the lawn costs £135.