Worksheet - Simultaneous equations, inequalities and polynomials

Year: Year 12 · 20

Show your working.

Question 1

Question 1 – Three kinds of simultaneous equations *(Section A · Skills practice)*

Solve each pair of simultaneous equations. Show all your working.

a. $4x - 3y = 15$ and $2x + 5y = 1$ [3 marks]
b. $y = 2x - 1$ and $x^2 + y^2 = 10$ [5 marks]
c. $xy = 12$ and $x + y = 7$ [4 marks]

Question 2

Question 2 – Linear and quadratic inequalities *(Section A · Skills practice)*

Solve each inequality and write the solution using inequality notation.

a. $5 - 3x \geq 20$ [2 marks]
b. $2(x + 4)$ [2 marks]
c. $x^2 - x - 12 > 0$ [3 marks]
d. $3x^2 \leq 12x$ [3 marks]

Question 3

Question 3 – Factorising a cubic *(Section A · Skills practice)*

Use the factor theorem to factorise each cubic completely.

a. $f(x) = x^3 - 7x + 6$ [4 marks]
b. $g(x) = 2x^3 + x^2 - 13x + 6$ [5 marks]

Question 4

Question 4 – Algebraic division and the remainder theorem *(Section A · Skills practice)*

$f(x) = x^3 + 2x^2 - 5x + 1$.

a. Divide $f(x)$ by $(x - 2)$, giving the quotient and the remainder. [3 marks]
b. Check the remainder using the remainder theorem. [1 mark]
c. Find the remainder when $f(x)$ is divided by $(x + 1)$. [2 marks]

Question 5

Question 5 – Finding unknown coefficients *(Section A · Skills practice)*

The polynomial $p(x) = x^3 + ax^2 + bx - 6$ has $(x - 2)$ as a factor. When $p(x)$ is divided by $(x + 1)$ the remainder is -12.

a. Write down two equations in a and b. [3 marks]
b. Solve them to find a and b. [3 marks]
c. Show that $p(x) = 0$ has only one real root. [3 marks]

Question 6

Question 6 – Combined inequalities *(Section A · Skills practice)*

Find the set of values of x that satisfy both inequalities in each case.

a. $2x + 1 > -5$ and $x^2 \leq 16$ [3 marks]
b. $x^2 - 5x + 4$ and $3x - 2 > 1$ [4 marks]

Question 7

Question 7 – Choosing a gym *(Section B · Real-life practice)*

Three gyms advertise their prices. Gym A charges £25 to join and then £18 per month. Gym B has no joining fee and charges £23 per month. Gym C charges £60 to join and then £14 per month.

Gym	Joining fee	Monthly fee
A	£25	£18
B	£0	£23
C	£60	£14

a. After how many months do gyms A and B cost the same? [3 marks]
b. Find the total cost of each gym for one year (12 months). [3 marks]
c. After how many whole months does gym C become cheaper than gym A? [3 marks]
d. Mia plans to stay for 6 months. Which gym is cheapest? [2 marks]

Question 8

Question 8 – The rectangular garden *(Section B · Real-life practice)*

A rectangular garden has perimeter 34 m and area 60 m^2. Let the length be l m and the width w m.

a. Write down two equations in l and w. [2 marks]
b. Solve them to find the dimensions of the garden. [4 marks]

c. A straight path joins opposite corners. Find its length. [2 marks]
d. Fencing costs £22 per metre. Work out the cost of fencing all four sides. [2 marks]

Question 9

Question 9 – Bakery profit *(Section B · Real-life practice)*

A bakery makes x hundred loaves a day. Its weekly profit model is $P = -0.5x^2 + 12x - 50$, where P is in hundreds of pounds.

a. Show that the bakery makes a profit when $x^2 - 24x + 100$. [2 marks]
b. Solve this inequality, giving the boundaries to 2 decimal places. [4 marks]
c. Find the number of loaves that gives the greatest profit and state that profit in pounds. [3 marks]
d. The bakery can bake between 600 and 2000 loaves. Between which production levels does it make a profit? [2 marks]

Question 10

Question 10 – Class trip budget *(Section B · Real-life practice)*

A school hires a coach for £420 and pays £8 per student for entry. The school budget is at most £1 000. The trip only runs with at least 40 students and the cost per student must not be more than £14. Let n be the number of students.

a. Show that the budget condition gives $n \leq 72$. [2 marks]
b. Show that the cost-per-student condition gives $n \geq 70$. [3 marks]
c. List the possible numbers of students. [1 mark]
d. For each possible value, work out the cost per student to 2 decimal places. [3 marks]

Question 11

Question 11 – A water jet on a hillside *(Section B · Real-life practice)*

A water jet leaves a nozzle at the origin and follows the path $y = 4x - x^2$, where x is the horizontal distance and y the height, both in metres. A hillside rises along the straight line $y = 0.5x$.

a. Find the coordinates of the point where the water hits the hillside. [4 marks]
b. Find the greatest height of the jet above the horizontal. [2 marks]
c. The vertical gap between the jet and the hillside at distance x is $g = 3.5x - x^2$. Find the greatest gap. [3 marks]
d. Find the horizontal distance at which the jet is 3 m above the hillside. [3 marks]

Question 12

Question 12 – Making an open box *(Section B · Real-life practice)*

A rectangular sheet of card is 20 cm by 14 cm. A square of side x cm is cut from each corner and the sides are folded up to make an open box.

a. Show that the volume is $V = 4x^3 - 68x^2 + 280x$. [3 marks]
b. Find V when $x = 2$. [1 mark]
c. The volume is 288 cm^3. Show that $x = 4$ is a solution and find the other valid solution to 2 decimal places. [6 marks]
d. State the range of values that x can take. [1 mark]

Question 13

Question 13 – Cinema and family tickets *(Section B · Real-life practice)*

At a cinema, 2 adult tickets and 3 child tickets cost £47.50. 3 adult tickets and 2 child tickets cost £52.50.

a. Form two simultaneous equations. [2 marks]
b. Solve them to find the price of an adult ticket and a child ticket. [4 marks]
c. A family has 2 adults and 4 children. Find the cost. [2 marks]
d. The cinema offers a family ticket for £50. Does this save money? [2 marks]

Question 14

Question 14 – Stopping distances *(Section B · Real-life practice)*

A simple model for the stopping distance d metres of a car travelling at v miles per hour is $d = 0.02v^2 + 0.6v$.

Speed v (mph)	20	30	40	50	60
Distance d (m)	20	36	56	80	108

a. Check the entries for 30 and 50 mph. [2 marks]
b. A school has a stopping-distance limit of 60 m. Show that v must satisfy $v^2 + 30v - 3000 \leq 0$. [2 marks]
c. Find the maximum speed to 1 decimal place. [4 marks]
d. Explain why the negative root is not used. [1 mark]

Question 15

Question 15 – Pantomime ticket prices *(Section C · Problem solving)*

A theatre sells adult tickets and child tickets for its pantomime. The takings on two nights are shown. Prices were the same on both nights.

Night	Adult tickets sold	Child tickets sold	Takings
Friday	150	130	£3 270
Saturday	200	120	£3 880

a. Let an adult ticket cost £a and a child ticket £c. Write two equations. [2 marks]
b. Solve to find a and c. [4 marks]
c. The hall seats 400. At least 150 seats must be sold to children. Find the greatest possible takings from one full night. [4 marks]
d. Each performance costs £2 900 plus £2.50 per ticket sold. How many tickets must be sold (all adult) to make a profit? [3 marks]

Question 16

Question 16 – A line and a curve *(Section C · Problem solving)*

The curve C has equation $y = x^2 - 4x + 1$ and the line L has equation $y = x + k$.

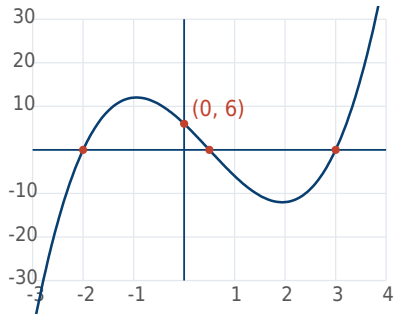
a. Show that the x-coordinates of the intersections satisfy $x^2 - 5x + 1 - k = 0$. [2 marks]
b. Find the range of k for which L meets C at two distinct points. [3 marks]
c. When $k = -5$ find the coordinates of the two intersections and the distance between them. [4 marks]
d. Find the value of k for which L is a tangent to C and the point of contact. [4 marks]

Question 17

Question 17 – Sketching a cubic *(Section C · Problem solving)*

$f(x) = 2x^3 - 3x^2 - 11x + 6$.

a. Show that $(x - 3)$ is a factor and factorise $f(x)$ completely. [4 marks]
b. Sketch the graph of $y = f(x)$, showing where it crosses the axes. [3 marks]
c. Solve $f(x) > 0$. [3 marks]
d. Find the remainder when $f(x)$ is divided by $(x + 1)$. [2 marks]



Question 18

Question 18 – An allotment problem *(Section C · Problem solving)*

A rectangular allotment has length $(x + 4)$ m and width $(x - 1)$ m. The area must be less than 50 m^2 and the perimeter must be at least 20 m.

x	3	4	5	6
Area (m^2)	14	24	36	50
Perimeter (m)	18	22	26	30

a. Check the table entries for $x = 4$. [2 marks]
b. Show that the area condition gives $x^2 + 3x - 54$ and solve it. [4 marks]
c. Show that the perimeter condition gives $x \geq 3.5$. [2 marks]
d. Find the range of x and list the whole-number dimensions that satisfy both conditions. [3 marks]

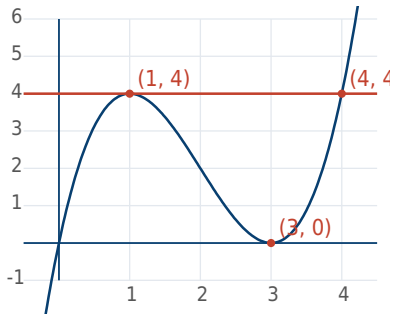
Question 19

Question 19 – Roller coaster *(Section C · Problem solving)*

Part of a roller coaster track is modelled by $y = x^3 - 6x^2 + 9x$ for $0 \leq x \leq 4$, where y is the height in tens of metres.

x	0	1	2	3	4
y	0	4	2	0	4

a. Show that $y = x(x - 3)^2$. What does this tell you about the point $(3, 0)$? [3 marks]
b. The ride has a horizontal safety beam at height $y = 4$. Show that the beam meets the track where $(x - 1)^2(x - 4) = 0$. [4 marks]
c. Explain what the repeated root at $x = 1$ means for the track. [2 marks]
d. For which values of x in the range is the track above ground level? [2 marks]



Question 20

Question 20 – Challenge: two factors of a quartic *(Section D · Challenge)*

$p(x) = x^4 + ax^3 - 7x^2 + bx + 6$. $(x - 2)$ and $(x + 3)$ are both factors of $p(x)$.

a. Find a and b. [5 marks]

b. Factorise $p(x)$ completely. [4 marks]
c. Solve $p(x)$. [3 marks]
d. Find the remainder when $p(x)$ is divided by $(x-3)$. [2 marks]