Test - Proof

Year: Year 12 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Proving with algebra

Prove that the sum of the squares of any two consecutive odd numbers is always 2 more than a multiple of 8.

a. Write down two consecutive odd numbers in terms of n. (1 mark)
b. Prove that the sum of their squares is always 2 more than a multiple of 8. (3 marks)

Question 2 (4 marks)

Question 2 – Counter-example

Show by a counter-example that each statement is false.

a. "If n is prime then $n + 2$ is prime." (2 marks)
b. "$(a + b)^2 = a^2 + b^2$ for all a, b." (2 marks)

Question 3 (4 marks)

Question 3 – Exhaustion

Prove that for any integer n, $n^2 + n$ is even, by considering the cases n even and n odd.

Question 4 (4 marks)

Question 4 – Proof by contradiction

Prove that there is no largest even number.

Question 5 (4 marks)

Question 5 – Problem: a table of seats

A hall has n rows. Row 1 has 10 seats and each row has 2 more seats than the row before.

a. Show that row n has $2n + 8$ seats. (1 mark)
b. Prove that the number of seats in every row is even. (1 mark)
c. Find the total number of seats in a hall with 15 rows. (2 marks)